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MULTIPLICITY AND CONCENTRATION BEHAVIOUR OF POSITIVE SOLUTIONS FOR SCHRÖDINGER-KIRCHHOFF TYPE EQUATIONS INVOLVING THE p-LAPLACIAN IN $\mathbb{R}^N$*

Huifang JIA (贾慧芳) *Gongbao LI* (李工宝)†
Hubei Key Laboratory of Mathematical Sciences and School of Mathematics and Statistics, Central China Normal University, Wuhan 430079, China
E-mail: *hf_jia@mails.ccnu.edu.cn; ligb@mail.ccnu.edu.cn*

Abstract In this article, we study the multiplicity and concentration behavior of positive solutions for the p-Laplacian equation of Schrödinger-Kirchhoff type

$$-\epsilon^p M\Big(\epsilon^{p-N}\int_{\mathbb{R}^N}|\nabla u|^p\Big)\Delta_p u + V(x)|u|^{p-2}u = f(u)$$

in $\mathbb{R}^N$, where Δ_p is the p-Laplacian operator, $1 < p < N$, $M : \mathbb{R}^+ \to \mathbb{R}^+$ and $V : \mathbb{R}^N \to \mathbb{R}^+$ are continuous functions, ϵ is a positive parameter, and f is a continuous function with subcritical growth. We assume that V satisfies the local condition introduced by M. del Pino and P. Felmer. By the variational methods, penalization techniques, and Lyusternik-Schnirelmann theory, we prove the existence, multiplicity, and concentration of solutions for the above equation.

Key words Schrödinger-Kirchhoff type equation; variational methods; multiple positive solutions; concentrating solution; penalization method

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1 Introduction

In this article, we consider the multiplicity and concentration behavior of positive solutions for the following Schrödinger-Kirchhoff type problem

$$\begin{cases} \epsilon^p M\Big(\epsilon^{p-N}\int_{\mathbb{R}^N}|\nabla u|^p\Big)(-\Delta)_p u + V(x)|u|^{p-2}u = f(u) & \text{in } \mathbb{R}^N, \\ u \in W^{1,p}(\mathbb{R}^N),\ u > 0 & \text{on } \mathbb{R}^N \end{cases} \qquad (Q_\epsilon)$$

involving the p-Laplacian, where $1 < p < N$, $M : \mathbb{R}^+ \to \mathbb{R}^+$, $V : \mathbb{R}^N \to \mathbb{R}^+$ are continuous function, ϵ is a small positive parameter, and $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian of u. We assume that the potential V satisfies

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†Corresponding author

(V_1) $V \in C(\mathbb{R}^N, \mathbb{R})$ and $\inf_{x\in\mathbb{R}^N} V(x) = V_0 > 0$;

(V_2) for each $\delta > 0$, there is an open and bounded set $\Lambda = \Lambda(\delta) \subset \mathbb{R}^N$ depending on δ such that

$$V_0 :< \min_{z\in\partial\Lambda} V(z),\ \Pi = \{x \in \Lambda : V(x) = V_0\} \neq \emptyset,$$

and

$$\Pi_\delta = \{x \in \mathbb{R}^N : \mathrm{dist}(x, \Pi) \leq \delta\} \subset \Lambda.$$

Problem (Q_ϵ) is of nonlocal because of the presence of the term $M(\int_{\mathbb{R}^N} |\nabla u|^p)$ which implies that the equation in (Q_ϵ) is no longer a pointwise identity.

Problem (Q_ϵ) is a natural extension of two classes of problems of great importance in applications, namely, Kirchhoff type problems and Schrödinger type problems.

(a) When $\epsilon = 1$, $p = 2$, and $V = 0$, problem (Q_ϵ) becomes the following problem

$$-M\Big(\int_{\mathbb{R}^N} |\nabla u|^2\Big)\Delta u = f(u) \quad \text{in} \quad \mathbb{R}^N, \tag{1.1}$$

which represents the stationary case of Kirchhoff model for small transverse vibrations of an elastic string by considering the effects of the changes in the length of the string during the vibrations.

(b) When $M \equiv 1$ and $p = 2$, (Q_ϵ) becomes

$$-\epsilon^2\Delta u + V(x)u = f(u) \ \text{ in } \quad \mathbb{R}^N, \tag{1.2}$$

which arises in different models, for example, to get a standing wave, that is, a solution of the form $\Psi(x, t) = \exp(-\mathrm{i}Et/\epsilon)u(x)$ of the following nonlinear Schrödinger equation

$$\mathrm{i}\epsilon\frac{\partial\Psi}{\partial t} = -\epsilon\Delta\Psi + (V(x) + E)\Psi - f(\Psi),\ \forall\ x \in \mathbb{R}^N, \tag{1.3}$$

where $f(t) = |t|^{s-2}t$, $N > 2$, and $2 < s < 2^* = \frac{2N}{N-2}$, and it will led to the study of (1.2). Many studies about the existence and concentration of positive solutions for problem (1.2) appeared in the past decade; see [1, 4, 16] and the references therein.

Recently, the following Kirchhoff type equation

$$\begin{cases} -\Big(a + b\displaystyle\int_{\mathbb{R}^3} |\nabla u|^2\Big)\Delta u + u = f(x, u) \text{ in } \mathbb{R}^3, \\ u \in H^1(\mathbb{R}^3) \end{cases} \tag{1.4}$$

has been studied extensively by many researchers, where $f \in C(\mathbb{R}^3 \times \mathbb{R}, \mathbb{R})$, and $a, b > 0$ are constants.

X. He and W. Zou in [7] studied (1.4) under the conditions that $f(x, u) := f(u) \in C^1(\mathbb{R}^+, \mathbb{R}^+)$ satisfies the Ambrosetti-Rabinowitz condition ((AR) condition in short):

$$\exists\mu > 4, 0 < \mu\int_0^u f(s)\mathrm{d}s \leq f(u)u,$$

$\lim_{|u|\to 0} f(u)/|u|^3 = 0$, $\lim_{|u|\to\infty} f(u)/|u|^q = 0$ for some $3 < q < 5$, and $f(u)/u^3$ is strictly increasing for $u > 0$, that is, $f(u)$ behaves like $|u|^{p-2}u$ $(4 < p < 6)$. They showed that the Mountain Pass Theorem and the Nehari manifold can be used directly to obtain a positive ground state solution to (1.4).