Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2018,**38B**(2):429–440
Mathematica Acta Scientia
数学物理学报
http://actams.wipm.ac.cn

QUALITATIVE ANALYSIS OF A STOCHASTIC RATIO-DEPENDENT HOLLING-TANNER SYSTEM*

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Abstract This article addresses a stochastic ratio-dependent predator-prey system with Leslie-Gower and Holling type II schemes. Firstly, the existence of the global positive solution is shown by the comparison theorem of stochastic differential equations. Secondly, in the case of persistence, we prove that there exists a ergodic stationary distribution. Finally, numerical simulations for a hypothetical set of parameter values are presented to illustrate the analytical findings.

Key words Stochastic ratio-dependent Holling-Tanner system; persistence in mean; stationary distribution

2010 MR Subject Classification 60J60; 60H10; 92D25

1 Introduction

The famous Holling-Tanner model which is also known as semi-ratio-dependent Holling type II for predator-prey interaction is expressed in the form:

$$\begin{cases} \dfrac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1-\dfrac{N(t)}{k}\right) - \dfrac{cN(t)}{a+N(t)}P(t), \\ \dfrac{\mathrm{d}P(t)}{\mathrm{d}t} = P(t)\left[s\left(1-h\dfrac{P(t)}{N(t)}\right)\right], \end{cases} \tag{1.1}$$

where $N(t)$ and $P(t)$ represent population densities of the prey and the predator, respectively. Parameters r, s, k, c, and a are positive constants, and represent the growth rate of prey

*Received May 4, 2016. This work is supported by NSFC of China Grant (11371085) and the Fundamental Research Funds for the Central Universities (15CX08011A).

and predator species, the carrying capacity of the prey, capturing rate, and half capturing saturation, respectively. Parameter $h > 0$ is the number of prey required to feed one predator at equilibrium conditions. Recently, more and more mathematicians and biologists have drawn attention on this model and have shown rich dynamical behaviors [1–7].

The traditional prey-predator models with the functional responses depend on prey density only, and are not demonstrated by the data of numerous experiments and observations [8–10]. In fact, the predator has to search and compete for food and the ratio-dependent function of the prey and the predator is more suitable to substitute for the model with complicated interaction between the prey and predator. Then, this model is expressed in the form:

$$\begin{cases} \dfrac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - \dfrac{N(t)}{k}\right) - \dfrac{cN(t)}{aP(t)+N(t)}P(t), \\ \dfrac{\mathrm{d}P(t)}{\mathrm{d}t} = P(t)\left[s\left(1 - h\dfrac{P(t)}{N(t)}\right)\right], \end{cases} \tag{1.2}$$

subjected to the same conditions as given above. Both theoretical and mathematical biologists studied the ratio-dependent Holling-Tanner model (refer to [8, 11–14]). In particular, Liang and Pan [12] analyzed system (1.2) and derived rich dynamical properties. Firstly, they nondimensionalized system (1.2) with the scaling $rt \to t$, $\frac{N}{k} \to N$, $\frac{cP}{rk} \to P$, and assumed the following conditions:

$(A_1) : \alpha\beta + 1 > \beta$;

$(A_2) : \alpha > 1$;

$(A_3) : (\alpha\beta + 2)\beta < (\delta\beta + 1)(\alpha\beta + 1)^2$;

$(A_4) : (\alpha\beta + 2)\beta > (\delta\beta + 1)(\alpha\beta + 1)^2$;

$(A_5) : \alpha\beta + 1 > \max\{\beta, \frac{1}{\delta}\}$,

where $\alpha = \frac{ra}{c}$, $\delta = \frac{sh}{c}$, $\beta = \frac{c}{hr}$. Then, the deterministic system (1.2) has the following results.

If condition (A_1) holds, then system (1.2) has a unique positive equilibrium $E^*(N^*, P^*)$, where $N^* = (1 - \frac{\beta}{\alpha\beta+1})$, $P^* = \beta N^*$; if condition (A_2) holds, then system (1.2) is permanent; if conditions (A_1) and (A_3) hold, then the positive equilibrium $E^*(N^*, \frac{P^*}{N^*})$ of system (1.2) is locally asymptotical state; if conditions (A_1) and (A_4) hold, then the positive equilibrium $E^*(N^*, \frac{P^*}{N^*})$ of system (1.2) is an unstable focus or node; If condition (A_5) holds, then the positive equilibrium $E^*(N^*, \frac{P^*}{N^*})$ of system (1.2) is globally asymptotical stable in the interior of the first quadrant; If conditions (A_1) and (A_4) hold, then system (1.2) has a unique limit cycle.

Most of the works on Holling-Tanner or ratio-dependent Holling-Tanner models are in a deterministic environment. May [15] pointed out that because of continuous fluctuations in the environment, the birth rates, the death rates, competition coefficients, and other parameters involved in model (1.2) exhibit random fluctuation to some extent. Recently, The effect of environment on dynamical behaviors of stochastic modified Holling-Tanner model was investigated by Ji [16, 17]. And for many other articles on the stochastic models, refer to [18–21]. In this article, we express system (1.2) with random perturbation in the following form:

$$\begin{cases} \mathrm{d}N(t) = N(t)\left(r - fN(t) - \dfrac{cP(t)}{aP(t)+N(t)}\right)\mathrm{d}t + \sigma_1 N(t)\mathrm{d}B_1(t), \\ \mathrm{d}P(t) = P(t)\left(s - \dfrac{mP(t)}{N(t)}\right)\mathrm{d}t + \sigma_2 P(t)\mathrm{d}B_2(t), \end{cases} \tag{1.3}$$