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CONTINUOUS FINITE ELEMENT METHODS FOR REISSNER-MINDLIN PLATE PROBLEM*

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Abstract On triangle or quadrilateral meshes, two finite element methods are proposed for solving the Reissner-Mindlin plate problem either by augmenting the Galerkin formulation or modifying the plate-thickness. In these methods, the transverse displacement is approximated by conforming (bi)linear macroelements or (bi)quadratic elements, and the rotation by conforming (bi)linear elements. The shear stress can be locally computed from transverse displacement and rotation. Uniform in plate thickness, optimal error bounds are obtained for the transverse displacement, rotation, and shear stress in their natural norms. Numerical results are presented to illustrate the theoretical results.

Key words Reissner-Mindlin plate; continuous element; triangle element; quadrilateral element; finite element method; uniform convergence

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1 Introduction

The Reissner-Mindlin plate model describes the deformation of a plate with thick to thin thickness, which is subject to a transverse load in terms of transverse displacement of the midplane and rotation of fibers normal to the midplane. In mixed methods, the shear stress is also taken as an independent variable, which plays a bridge role in the finite element analysis of the Reissner-Mindlin plate problem.

As we know, it is important how to avoid shear locking phenomenon and boundary -layer effects in designing finite element methods for solving the Reissner-Mindlin plate problem; see [1, 2]. Mathematically, it amounts to how to establish convergence independent of the thickness of the plate. This independence strongly relies on the uniform smoothness of the shear stress with respect to the plate-thickness. Therefore, how to approximate the shear stress is critical.

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There have been substantial interest in mixed finite element methods or finite element displacement methods, and many articles were available in the literature. Among others, Hu, Ming, and Shi studied several important and useful quadrilateral nonconforming elements; see [3–15]. Hybrid elements of practical interest are studied by Xie et al; see [16–21]. Some stabilized nonconforming elements are analyzed by Duan; see [22–26].

With certain projection operators, the shear stress can be well-approximated, such as $H(\mathrm{curl};\Omega)$ local interpolation, $H(\mathrm{div};\Omega)$ (or $(H^1(\Omega))^2$) global L^2 projection, $(L^2(\Omega))^2$ local L^2 projection, etc; see [22, 23]. Some times, if necessary, the Galerkin variational formulation may be modified, as well as the plate-thickness, such as discontinuous Galerkin method, first-order least-squares method, etc; see [27–30] and references therein. These modifications are generally devoted to the stability of the transverse displacement and the convergence of the shear stress. Moreover, using unstandard finite elements for the shear stress can also yield locking-free (even boundary-layer-effects-free) approximating solutions; for example, see [31], etc. Due to locking phenomenon, to develop efficient fast solvers are important; for example, see [32], etc.

In this article, we propose two new finite element methods to solve the Reissner-Mindlin plate problem. Different from the above literature, we employ continuous elements for transverse displacement and rotation, while shear stress is still element-locally computed. In these methods, the approximating spaces and the norms for the shear stress are important to derive optimal error bounds uniformly in the plate-thickness, in particular for the case of quadrilaterals.

In the first method, the transverse displacement is approximated by conforming (bi)linear macro-elements, and the rotation by conforming (bi)linear elements enriched by local bubble functions. Moreover, the Galerkin formulation is augmented to enhance the stability of the transverse displacement and a local L^2 projection operator is introduced, because of the gradient of the space for this variable being not a subspace of that for the shear stress.

For triangles, we choose the space of discontinuous piecewise constant polynomials for the shear stress, and for quadrilaterals we choose the same space but carrying a local given function. Thus, transforming this method into a mixed form, we show that the method is uniformly stable and uniformly optimally convergent.

In the second method, the transverse displacement is approximated by conforming (bi)quadratic elements, and the rotation by conforming (bi)linear elements. Moreover, the plate-thickness is locally modified. Not any reduction operator is needed, and what choices for the approximating space for the shear stress is unnecessary to consider in practical computations.

For triangles, we choose the space of discontinuous piecewise linear elements for the shear stress, and for quadrilaterals we choose the space which may be viewed as certain variant of the first-order Raviart-Thomas rectangular element also carrying the same local function as in the first method. Thus, transforming this method into a mixed form, we also obtain optimal error bounds, uniform in the plate-thickness.

When analyzing these two methods, the key point for the first method is that we can control the jumps of the normal components of the shear stress across interelement boundaries; while that for the second method is that the transverse displacement may be split as two parts which have different regularities with respect to the plate-thickness and this split make it unnecessary