



SOLUTIONS TO BSDES DRIVEN BY BOTH FRACTIONAL BROWNIAN MOTIONS AND THE UNDERLYING STANDARD BROWNIAN MOTIONS*

Yuecai HAN (韩月才)

*Department of Mathematical Finance, School of Mathematics, Jilin University,
Changchun 130012, China
E-mail: hanyc@jlu.edu.cn*

Yifang SUN (孙一芳)[†]

*Department of Probability and Mathematical Statistics, School of Mathematics, Jilin University,
Changchun 130012, China
E-mail: syf15@mails.jlu.edu.cn*

Abstract The local existence and uniqueness of the solutions to backward stochastic differential equations (BSDEs, in short) driven by both fractional Brownian motions with Hurst parameter $H \in (1/2, 1)$ and the underlying standard Brownian motions are studied. The generalization of the Itô formula involving the fractional and standard Brownian motions is provided. By theory of Malliavin calculus and contraction mapping principle, the local existence and uniqueness of the solutions to BSDEs driven by both fractional Brownian motions and the underlying standard Brownian motions are obtained.

Key words Backward stochastic differential equations; malliavin calculus; fractional Brownian motions; Itô formula

2010 MR Subject Classification 60H07; 60H20

1 Introduction

Let $(\Omega, \mathcal{F}, P)$ be a basic probability space. $T \in (0, +\infty)$ is a finite time to be confirmed. Let $B^H = (B_1^H(t), \dots, B_m^H(t))$, $t \in [0, T]$ be an m -dimensional fractional Brownian motion (fBm, for short) of Hurst parameter $H \in (1/2, 1)$. It is allowed that the Hurst parameter H may be different from $B_i^H(t)$ to $B_j^H(t)$ except notation for each $i \neq j$. In other words, $B_j^H(t)$, $j = 1, \dots, m$, are independent, continuous, and mean zero Gaussian processes with the following covariance

$$\mathbb{E}[B_i^H(t)B_j^H(s)] = \frac{1}{2}\delta_{ij} \left(|t|^{2H} + |s|^{2H} - |t-s|^{2H} \right),$$

*Received November 28, 2016; revised May 18, 2017. This work was supported by NSFC grant (11371169), and China Automobile Industry Innovation and Development Joint Fund (U1564213).

[†]Corresponding author

where δ is the Kronecker symbol. For each $j = 1, \dots, m$, and $t \in [0, T]$, let $\mathcal{F}_t^j$ be the σ -algebra by $B_j^H(s)$, $0 \leq s \leq t$, augmented by all the P -zero measurable events. We denote the corresponding filtration by $\mathbb{F} = \{\mathcal{F}_t, 0 \leq t \leq T\} = (\mathcal{F}_t^1, \dots, \mathcal{F}_t^m)_{0 \leq t \leq T}$. Let $W = (W_1(t), \dots, W_m(t))$, $t \in [0, T]$ be an $\mathbb{F}$ -adapted m -dimensional standard Brownian motion (sBm, for short). We say that $W = (W_1(t), \dots, W_m(t))$ is the underlying standard Brownian motion corresponding to $B^H = (B_1^H(t), \dots, B_m^H(t))$, $t \in [0, T]$ if

$$B_j^H(t) = \int_0^t Z_H(t, s) dW_j(s), \quad \forall t \in [0, T], \quad j = 1, \dots, m,$$

where

$$Z_H(t, s) = (H - \frac{1}{2})\kappa_H s^{\frac{1}{2}-H} \int_s^t u^{H-\frac{1}{2}}(u-s)^{H-\frac{3}{2}} du,$$

and $\kappa_H = \sqrt{\frac{2H\Gamma(\frac{3}{2}-H)}{\Gamma(\frac{1}{2}+H)\Gamma(2-2H)}}$.

We consider the following backward stochastic differential equation

$$Y_t = \xi + \int_t^T f(r, Y_r, Z_r) dr + \int_t^T g(r, Y_r, Z_r) dB_r^H - \int_t^T Z_r dW_r, \quad (1.1)$$

where ξ is given terminal value, f and g are the given (random) generators. To solve this equation is to find a pair of adapted processes $(Y_t, Z_t)_{0 \leq t \leq T}$, satisfying the above equation (1.1).

Backward stochastic differential equations have a variety of applications in fields such as stochastic optimal control theory, mathematical finance, probability interpretation of solutions of quasi-linear partial differential equations and so on. The general nonlinear backward stochastic differential equations with respect to standard Brownian motions were first introduced by Pardoux and Peng [25]. Peng recently gave a survey [23] on the developments in the theory of nonlinear BSDEs during the past 20 years, including the existence and uniqueness of the solutions, nonlinear Feynman-Kac formula, nonlinear expectation and many other results in BSDEs theory and their applications to dynamic pricing and hedging in an incomplete financial market.

Because the fractional Brownian motions are not semimartingales except the case $H = 1/2$, it can be used to describe and explain some natural or society phenomena, such as to model hydrology, climatology, signal processing, network traffic analysis, control theory, finance as well as various other fields. This makes the stochastic analysis for fractional Brownian motions challenging and fascinating. In recent years, the stochastic integral with respect to the fBm had been investigated from different perspectives by many authors [1, 9, 10, 20, 26], and so on. For the Malliavin calculus with respect to fBm and applications, thanks to the contribution of [8, 14, 21, 22] and references there. Hu and Peng in [17] had studied the backward stochastic differential equations driven by fractional Brownian motions. They studied the general and linear BSDEs driven by fractional Brownian motions. Shortly afterwards Fei, Xia, and Zhang in [11] had analogous results as Hu and Peng in [17] to the BSDEs driven by both standard and fractional Brownian motions. In both articles, the pair of solutions of BSDEs $(Y_t, Z_t; 0 \leq t \leq T)$ relied on the simple process $\eta_t = \eta_0 + b(t) + \int_0^t \sigma(t) dB_t^H$, where b and σ are determinate function in $L^2([0, T])$. In this article, we will cancel this dependency.

Especially, in stochastic optimal control theory for controlled system, BSDEs as the adjoint equations corresponding to the state equations to describe optimal control problem are essential,

Download English Version:

<https://daneshyari.com/en/article/8904425>

Download Persian Version:

<https://daneshyari.com/article/8904425>

[Daneshyari.com](https://daneshyari.com)