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STABILITY AND BIFURCATION ANALYSIS OF A DELAYED INNOVATION DIFFUSION MODEL*

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Abstract In this article, a nonlinear mathematical model for innovation diffusion with stage structure which incorporates the evaluation stage (time delay) is proposed. The model is analyzed by considering the effects of external as well as internal influences and other demographic processes such as emigration, intrinsic growth rate, death rate, etc. The asymptotical stability of the various equilibria is investigated. By analyzing the exponential characteristic equation with delay-dependent coefficients obtained through the variational matrix, it is found that Hopf bifurcation occurs when the evaluation period (time delay, τ) passes through a critical value. Applying the normal form theory and the center manifold argument, we derive the explicit formulas determining the properties of the bifurcating periodic solutions. To illustrate our theoretical analysis, some numerical simulations are also included.

Key words Innovation diffusion model; stability analysis; Hopf-bifurcation; normal form theory; center manifold theorem

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1 Introduction

Articles [1, 2], and in particular [3], represented the starting point of a stream concerning the study of the time path of the spread of new consumer durables over a population of potential consumers. The first marketing innovation diffusion model was proposed by Bass, and it is still providing the foundation for developing the new hypotheses for gaining insight into the diffusion

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and penetration of innovations among the potential adopters. In the Bass model, the market m is assumed to be fixed, and the evolution of the adopters is described by

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = p[m - A(t)] + \frac{q}{m}A(t)[m - A(t)].$$

Where $A(t)$ is the number of adopters at time t; The parameter p takes into account new adopters who join the market as a result of external influences such as activities of firms in the market, advertising, attractiveness of the innovation; The parameter q is the coefficient of imitation and refers to the rate of influence of adopters on nonadopters.

The model has been widely used to understand the spread of new products and to understand the underlying phenomena responsible for the diffusion of new technology(market innovation) or new products in the market [4–13]. According to Rogers [14], "a technological innovation is a design for instrumental action that reduces the uncertainty in the cause-effect relationships involved in achieving the desired outcome." The literature on mathematical modeling of innovation diffusion has accounted for a large number of contributions with different directions of investigation since the early 1960s. Rogers summarized his previous research in his book and concluded that the diffusion of innovations consists of five steps: awareness (individual is exposed to innovation), interest (individual seeks more information), evaluation (individual applies innovation to his or her situation), trial (individual uses innovation on a small scale), and adoption (individual makes full use of the innovation) [14]. Furthermore, through the behavior research, he found that the five steps innovation diffusion processes can be simplified into two-step flow process, that is, media influences innovative opinion leaders to adopt a new product, who in turn force other people to adopt the product as well [14–18]. Mahajan et al [16] discussed all the earlier contributions of the management and marketing literature to the cumulative understanding of the innovation diffusion dynamics.

The models with a time delay have been proposed by many researchers which exhibits the evaluation stage of a product [19–22]. A mathematical model is intended to describe the dynamics of users of one product in two different patches [23]. Stability of competitive innovation diffusion model in a market is explained in [24, 25]. Global stability of an innovation diffusion model for n products is discussed in [26]. An innovation diffusion model with the nonlinear acceptance is reviewed to describe the dynamics of three competing products in a market [27]. A binomial innovation diffusion model for a variable size market for demographic processes of entrance-exit from each market compartment was studied by F. Centrone [28]. Another model with time delay is developed to consider the evaluation stage. Shukla et al proposed that innovation diffusion process is affected by variable external influences such as advertisements as well as the change of density of nonadopters population because of intrinsic growth rate, emigration or death rate, etc. The model analysis shows that the adopter's population density increases as the parameters related to the growth rate of nonadopters population as well as the rate of external influences increase. The main effect of increasing density of variable external forces is to make the equilibrium level of adopters population density reach to its equilibrium with a much faster rate [29]. Wang et al [22] have considered the aspect of delay in their studies to describe the process of evaluation and decision-making whereas Shukla et al explained that the innovation diffusion process is affected by variable external influences such as advertisements as well as change of density of non-adopters population because of intrinsic growth rate, emi-