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INITIAL-BOUNDARY PROBLEM FOR THE 1-D EULER-BOLTZMANN EQUATIONS IN RADIATION HYDRODYNAMICS*

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Abstract We study the initial-boundary value problem for the one dimensional Euler-Boltzmann equation with reflection boundary condition. For initial data with small total variation, we use a modified Glimm scheme to construct the global approximate solutions $(U_{\triangle t,d}, I_{\triangle t,d})$ and prove that there is a subsequence of the approximate solutions which is convergent to the global solution.

Key words global existence; Euler-Boltzmann equations; initial-boundary; Glimm scheme

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1 Introduction

Consider the one-dimensional Euler-Boltzmann equations in Ω,

$$\begin{cases} \rho_t + (\rho v)_x = 0, & (1.1) \\ (\rho v)_t + (\rho v^2 + P)_x = -\mathcal{S}_F, & (1.2) \\ (\rho E)_t + ((\rho E + P)v)_x = -\mathcal{S}_E, & (1.3) \\ \dfrac{1}{c} I_t + \omega I_x = \mathcal{S}, & (1.4) \end{cases}$$

where $(x,t) \in \Omega$, ρ is the density, v is the velocity of gas, c is the speed of light ($v^2 \ll c^2$), the total energy

$$E = \frac{1}{2}v^2 + e, \tag{1.5}$$

where e is the internal energy, $P = P(\rho, S)$ is the pressure, and S stands for the entropy of gas, $I = I(x,t;\nu,\omega)$ is the radiative intensity which depends on two extra variables: the radiative frequency $\nu \in [0,+\infty)$ and the radiative angular $\omega \in [-1,1]$. The domain Ω is given by

$$\Omega = \{(x,t) |\, x \geq 0, t \geq 0\}. \tag{1.6}$$

The first three equations (1.1)–(1.3) are Euler equations coupled with the source terms. The last equation (1.4) is the radiative transfer equation, it describes the motion of photons

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which are massless particles and transferring with the speed of light c. The coupled problem requires us to consider the effects of radiation to the hydrodynamic system.

The radiative energy source $\mathcal{S}_E$ and the radiative force $\mathcal{S}_F$ are defined as follows :

$$\mathcal{S}_F := \frac{1}{c}\int_0^{+\infty}\int_{-1}^{1} \omega \mathcal{S}(x,t,\nu,\omega)\,\mathrm{d}\omega\,\mathrm{d}\nu, \tag{1.7}$$

$$\mathcal{S}_E := \int_0^{+\infty}\int_{-1}^{1} \mathcal{S}(x,t,\nu,\omega)\,\mathrm{d}\omega\,\mathrm{d}\nu. \tag{1.8}$$

The radiation source is splited into two parts, that is

$$\mathcal{S} = \mathcal{S}_{a,e} + \mathcal{S}_s, \tag{1.9}$$

here, $\mathcal{S}_{a,e}$ describes the absorption-emission contribution, and $\mathcal{S}_s$ describes scattering contribution defined as follows,

$$\begin{cases} S_{a,e}(x,t,\nu,\omega) := \sigma_a(t,\nu,P,T)[B(\nu,T) - I(x,t,\nu,\omega)], & (1.10)\\ \mathcal{S}_s(x,t,\nu,\omega) := \sigma_s(t,\nu,P,T)[\tilde{I}(x,t,\nu) - I(x,t,\nu,\omega)], & (1.11)\end{cases}$$

where $\sigma_a(t,\nu,P,T)$ and $\sigma_s(t,\nu,P,T)$ are positive and bounded, and

$$\tilde{I}(x,t,\nu) := \int_{-1}^{1} I(x,t,\nu,\omega)\,\mathrm{d}\omega. \tag{1.12}$$

In this paper, let $U := (\rho, v, \mathcal{S})^T$, we consider (1.1)–(1.4) with the initial condition,

$$U|_{t=0}(x,t) = \begin{cases} U_0(x), & \text{if } 0 < x \le N,\\ U_\infty, & \text{if } x > N,\end{cases} \tag{1.13}$$

$$I|_{t=0}(x,t,\nu,\omega) = \begin{cases} I_0(x,\nu,\omega), & \text{if } 0 < x \le N,\\ I_\infty(\nu,\omega), & \text{if } x > N,\end{cases} \tag{1.14}$$

for some $N > 0$, where U_0, I_0 are measurable functions, U_∞ is a constant state, $I_\infty = B(\nu, T_\infty)$ where T_∞ is associated with U_∞, and the function $B(\nu,T)$ describes the equilibrium state,

$$B(\nu,T) = 2h\nu^3 c^{-2}(\mathrm{e}^{\frac{h\nu}{k_B T}} - 1)^{-1}. \tag{1.15}$$

Obviously, (U_∞, I_∞) is a solution of (1.1)–(1.4) by applying the initial conditions.

Meanwhile, the coupled problem satisfies the following boundary condition:

$$\begin{cases} v|_{x=0} = 0, \ \ t > 0 & (1.16)\\ I|_{x=0} = I(x=0,t,\nu,-\omega) + b(t;\nu,-\omega), \ \ \omega \ge 0,\ \nu \ge 0,\ t \ge 0. & (1.17)\end{cases}$$

The function $b(t;\nu,\omega)$ defined in $C([0,+\infty)\times[0,+\infty)\times[-1,1])$ is supposed to satisfy

$$\int_0^{+\infty}\int_{-1}^{1} T.V.|b(\cdot;\nu,\omega)|\,\mathrm{d}\omega\,\mathrm{d}\nu << 1, \tag{1.18}$$

$$\int_0^{+\infty}\int_{-1}^{1}\int_0^{+\infty} |b(t;\nu,\omega)|\,\mathrm{d}t\,\mathrm{d}\omega\,\mathrm{d}\nu < +\infty. \tag{1.19}$$

Our goal in this paper is to prove the existence of global BV solution for the coupled system (1.1)–(1.4) with above given initial date (1.13)–(1.14) and boundary condition (1.16)–(1.17). There are many works devoted to the Cauchy problem for Euler equations with radiation such