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THE OPTIMAL CONTROL FOR PROMOTING THE COOPERATION IN EVOLUTION GAME GENERATED BY PRISONER'S DILEMMA*

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Abstract Natural selection opposes the evolution of cooperation unless specific mechanisms are at work in Prisoner's Dilemma. By taking advantage of the modern control theory, the controller design is discussed and the optimal control is designed for promoting cooperation based on the recent advances in mechanisms for the evolution of cooperation. Two control strategies are proposed: compensation control strategy for the cooperator when playing against a defector and reward control strategy for cooperator when playing against a cooperator. The feasibility and effectiveness of these control strategies for promoting cooperation in different stages are analyzed. The reward for cooperation can't prevent defection from being evolutionary stable strategy (ESS). On the other hand, compensation for the cooperator can't prevent defection from emerging and sustaining. By considering the effect and the cost, an optimal control scheme with constraint on the admissible control set is put forward. By analyzing the special nonlinear system of replicator dynamics, the exact analytic solution of the optimal control scheme is obtained based on the maximum principle. Finally, the effectiveness of the proposed method is illustrated by examples.

Key words evolutionary games; Prisoner's Dilemma; evolution of cooperation; promoting cooperation control; optimal control

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1 Introduction

Cooperation is a favorite topic among evolutionary biologists because it seems to be at variance with natural selection. Why should one individual help another who is a potential competitor in the struggle for survival? This is a lasting puzzle. A definition for games that are cooperative dilemmas was proposed [1]. In the Prisoner's Dilemma, no matter what the other person does, it is best for one to defect. This is the dilemma: rational players who act in order to maximize their payoffs. Mutual cooperation leads to a higher payoff than mutual defection, but cooperation is irrational. The Prisoner's Dilemma represents the most stringent situation, where natural selection opposes cooperation unless a mechanism for the evolution of

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cooperation is at work. A mechanism for the evolution of cooperation is an interaction structure, specifying how the individuals of a population interact to accumulate payoff and to compete for reproduction. Nowak et al. [2] proposed that interaction structures can be classified into five mechanisms: direct reciprocity [3, 4], indirect reciprocity [5, 6], network reciprocity [7–9], group selection [10, 11], kin selection [12] and so on.

When the individual strategy is not from the genetic or given rules, but can be determined by the individual as feedback of information of the game, the evolution of game dynamics can be studied from the perspective of engineering design, where the individual strategy is regarded as control [13, 14]. Wang et al. [15] investigated the effects of coevolutionary dynamics on the evolution of cooperation. In [15], the state of the system was described by the collective level of individual strategies and coevolved with individual properties, constituting a feedback mechanism which could promote cooperation under appropriate time scale of strategy updating to game happening. Liang et al. [16] proved that the proportion of cooperators could be manipulated as the stochastically stable equilibrium which was being shifted by changing the game parameters. So the results indicated a promising approach to controlling the proportion of cooperators in large populations. Punishing the defectors or rewarding the cooperators is also an important way to promote cooperation [17]. For public goods game, Tatsuya et al. [18] revealed that even with compulsory participation, rewards could maintain cooperation within an infinitely large population. Tatsuya et al. [18] designed the incentive mechanism for public funds: before game started, the participants must pay a fixed amount of donation to form an incentive fund to reward cooperative behavior in the game. However, in [13–18], the control designers are game participants, and the behavior of control is generated within the game group. In the existing literatures, the researches of the mechanism of cooperation evolution of biological groups mostly focused on a relatively stable external environment, more precisely, the cooperation evolution is determined by the individual properties, the relationships between individuals and the parameters within the group. But few people consider the external factors of the group. In the literatures of considering economic or social problems, the governments are always treated as game participants.

In fact, a central authority can become a promoter of a “nudge policy”. The cooperation between enterprises is often caught in the Prisoner’s Dilemma in the industry agglomeration district just formed soon, then Park Management Committee as an external controller should influence a population to achieve a cooperative social norm. In view of the problem of the complicate dynamic game in the process of coal mining safety supervision and regulation in China, Liu et al. [19] analyzed the systemically evolutionary game and proposed the dynamic penalty-incentive measure to control the fluctuations and present an ideal evolutionary stable strategy (ESS) under the condition that coal enterprises would choose safety production as their optimal strategy. For the implementation of environmental regulation in China, Pan et al. [20] analyzed the evolutionary process of decision among local government, enterprises and central government from the perspective of evolutionary game theory. The results showed that the environmental regulation strategy of the local government was affected by the weight coefficient of environmental quality index, the cost of implementation of environmental regulation, the punishment of central government to local government, and so on. On the robustness of cooperative social norms for fishery extraction, Tilman et al. [21] provided a useful example of