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DISCRETENESS OF THE EXTERIOR TRANSMISSION EIGENVALUES*

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Abstract In this paper we consider a kind of exterior transmission problem in which the refractive index $n(x)$ is a piecewise positive constant. Through establishing an equivalent boundary integral system, we obtain that the set of exterior transmission eigenvalues is a discrete set. Furthermore, we prove that there does not exist a transmission eigenvalue under some conditions.

Key words exterior transmission eigenvalue; inverse scattering problem; Helmholtz equation

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1 Introduction

Transmission eigenvalue problem plays a central role in the qualitative approach to inverse scattering theory (see [1]). In the past few years transmission eigenvalues have become an important area of research in inverse scattering theory (see [2] and the references therein). The transmission eigenvalue problem is a new class of boundary value problems for elliptic equations, but this problem is not covered by the standard theory of elliptic partial differential equation since as it stands it is neither elliptic nor self-adjoint. Of particular interest is the spectrum associated with this boundary value problem, more specifically the properties of eigenvalues which are called transmission eigenvalues.

Roughly speaking, transmission eigenvalue problems include two different problems, that is, the interior transmission problem and the exterior transmission problem. The interior transmission eigenvalue problem which arises naturally in the scattering of incident plane waves by a bounded non-absorbing media has by far been the most extensively studied (see [2–9, 13, 17, 19]). More related research works can be found in [10, 11, 15, 18, 20] and the references therein. However, the exterior transmission eigenvalue problem is of much more recent origin [6, 12] and [21] and arises when one considers the scattering of point sources situated

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in a cavity that is bounded by a penetrable non-absorbing medium of compact supports and seek to determine the shape of the cavity from internal measurements.

In this paper we consider a kind of exterior transmission problem in which the refractive index $n(x)$ is a piecewise positive constant. We focus on the discreteness of the exterior transmission eigenvalues. As we known, existence of the exterior transmission eigenvalues is an attractive and difficult question (see [12]), so in this paper we just give a nonexistence result, that is, there does not exist a transmission eigenvalue under some conditions.

Based on the idea of [14] and [15], we assume that the refractive index $n(x)$ has two different constants n_1 and n_2 in $D_1\backslash\bar{D}$ except $n = 1$ in the domain $R^3\backslash\bar{D}_1$. Complicated calculation shows that the original problem (2.1) with (2.2) and (2.3) is equivalent to a boundary integral system. In addition, we find that n_1 and n_2 are not arbitrary constants when we want to obtain the uniqueness of the solution to the problem (2.1) with (2.2) and (2.3), and n_1 and n_2 should satisfy some condition, for example, $n_1 - 1 = 1 - n_2$. The main challenge is to establish an equivalent boundary integral system (see (2.25)), and show the uniqueness of the solution to the problem (2.1) with (2.2) and (2.3) under some suitable condition.

The plan of this paper is as follows. In the next section we introduce the exterior transmission problem, and establish an equivalent boundary integral system. By the equivalent boundary integral system we obtain the discreteness result in Section 3, and a nonexistence result is also given in Section 3.

2 The Exterior Transmission Eigenvalue Problem

Let $D \in R^3$ and $D_2 \in R^3\backslash\overline{D}$ be simply connected bounded regions of R^3 with C^2 boundary ∂D and ∂D_2. Let D_1 be a bounded simply connected region containing D and D_2 in its interior with C^2 boundary ∂D_1 and denote by ν be the unit outward normal to ∂D, ∂D_1 and ∂D_2 respectively. The refractive index $n = n_1$ in $D_1\backslash\left(\overline{D}\cup\overline{D}_2\right)$, $n = n_2$ in D_2, $n = 1$ in $R^3\backslash\overline{D}_1$, and n_1, n_2 are positive constants (see the Figure 1).

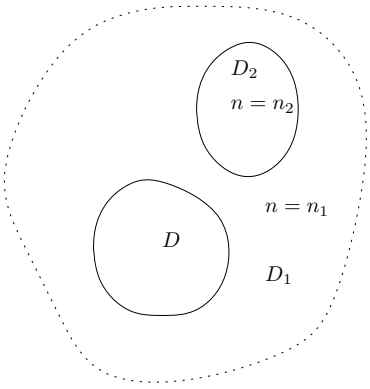

Figure 1 The exterior transmission eigenvalue problem