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EXISTENCE OF THE UNIFORM TRAJECTORY ATTRACTOR FOR A 3D INCOMPRESSIBLE NON-NEWTONIAN FLUID FLOW*

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Abstract This paper studies the trajectory asymptotic behavior of a non-autonomous incompressible non-Newtonian fluid in 3D bounded domains. In appropriate topologies, the authors prove the existence of the uniform trajectory attractor for the translation semigroup acting on the united trajectory space.

Key words incompressible non-Newtonian fluid; uniform trajectory attractor; topological space

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1 Introduction

In the theory of fluid dynamics, the motion of the incompressible n-dimensional fluid flows can be described by the systems (see [25])

$$\frac{\partial u}{\partial t}+(u\cdot\nabla)u+\nabla\cdot\mathbb{T}(e(u))=g(x,t), \tag{1.1}$$

$$\nabla\cdot u=0, \tag{1.2}$$

where $u=(u_1,u_2,\cdots,u_n)$ is the velocity field of the fluid, $g(x,t)$ is the external force, $\nabla=(\partial/\partial x_1,\partial/\partial x_2,\cdots,\partial/\partial x_n)$ is the gradient operator; $e(u)=(e_{ij}(u))_{n\times n}$ means the rate of the deformation tensor with

$$e_{ij}(u)=\frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\Big),\ \ i,j=1,2,\cdots,n. \tag{1.3}$$

$\mathbb{T}(e(u))=(\mathbb{T}_{ij}(e(u)))_{n\times n}$, called the constitutive relation of the fluid, is a transformation from the set of $n\times n$ symmetric matrixes $\mathbb{M}_{n\times n}$ to $\mathbb{M}_{n\times n}$, such that

$$\mathbb{T}_{ij}(e(u))=p\delta_{ij}-\mathbb{T}^u_{ij}(e(u)),\ \ i,j=1,2,\cdots,n. \tag{1.4}$$

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In (1.4), the scalar function p is the pressure, $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ otherwise, $\mathbb{T}^u_{ij}(e(u)) : \mathbb{M}_{n\times n} \longmapsto \mathbb{R}$ is the extra stress tensor of the fluid which is a function of $e(u)$. The incompressibility of the fluid is described by $\nabla \cdot u = 0$.

According to the constitutive relation, incompressible fluids can be divided into two types: Newtonian and non-Newtonian. If the dependence of the extra stress tensor $\mathbb{T}^u = (\mathbb{T}^u_{ij})_{n\times n}$ on the deformation tensor $e(u)$ is linear, for example,

$$\mathbb{T}^u(e) = 2\nu e, \text{ where } \nu \text{ is a positive constant,} \tag{1.5}$$

then the fluid is said to satisfy the Stokes law (see [25, p.13]) and is called the Newtonian fluid. Generally speaking, gases, water, motor oil, alcohols, and simple hydrocarbon compounds tend to be Newtonian and their motions can be described by the Navier-Stokes (NS) equations. A fluid whose extra stress tensor cannot be adequately described by (1.5) is usually called to be non-Newtonian. For instance, molten plastics, polymer solutions and paints tend to be non-Newtonian fluids.

For a class of non-Newtonian fluids in the three-dimensional (3D) space, the authors of [4] introduced the following constitutive relation

$$\mathbb{T}_{ij}(e(u)) = -p\delta_{ij} + 2\mu_0(\varepsilon + |e(u)|^2)^{-\alpha/2}e_{ij} - 2\mu_1\Delta e_{ij}, \ \ i,j = 1,2,3, \tag{1.6}$$

where ε, μ_0, μ_1 and $\alpha > 0$ are positive constitutive parameters and

$$|e(u)|^2 = \sum_{i,j=1}^{3} |e_{ij}(u)|^2. \tag{1.7}$$

If we set

$$\mu(u) = 2\mu_0(\varepsilon + |e(u)|^2)^{-\alpha/2}, \tag{1.8}$$

then the relations (1.1)–(1.3), (1.6) and (1.8) lead to the following nonlinear equations for a class of isothermal and incompressible non-Newtonian fluids

$$\frac{\partial u}{\partial t} + (u\cdot\nabla)u - \nabla\cdot\big(\mu(u)e(u) - 2\mu_1\Delta e(u)\big) + \nabla p = g(x,t), \tag{1.9}$$

$$\nabla\cdot u = 0. \tag{1.10}$$

The above fluids also are called the bipolar fluids, which were introduced by Nečas et al. as a regularization of the so-called p-fluids, see [4].

The non-Newtonian fluids equations (1.9)–(1.10) are widely applied in the real world and extensively studied. There are many papers on the existence, uniqueness, regularity and long time behavior of solutions to the initial boundary value problem (IBVP) or to its related versions (see e.g. [2, 6, 7, 15–21, 23, 25, 28, 31–39]). For example, [6] proved the existence and uniqueness of solutions in two-dimensional (2D) unbounded channels; [20] investigated the partial regularity of suitable weak solution; [28] established the existence, uniqueness and regularity of solutions to the Cauchy problem; [7, 23, 31, 33, 34, 36] studied the existence, regularity and related properties of attractors. Especially, in the autonomous case, [33] proved the existence of trajectory attractor and global attractor for equations (1.9)–(1.10) in 2D bounded domains.

We know that, when $\alpha > 0$, the solutions of equations (1.9)–(1.10) can be non-unique, even in the 2D (bounded or unbounded) case (see [6, 33]). There are three approaches introduced to overcome the difficulties associated to possible non-uniqueness of solutions in the study of