Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2018,**38B**(1):220–236
Mathematica Acta Scientia 数学物理学报
http://actams.wipm.ac.cn

ASYMPTOTIC EQUIVALENCE OF ALTERNATELY ADVANCED AND DELAYED DIFFERENTIAL SYSTEMS WITH PIECEWISE CONSTANT GENERALIZED ARGUMENTS*

Kuo-Shou CHIU
Departamento de Matemática, Facultad de Ciencias Básicas, Universidad Metropolitana de Ciencias de la Educación, José Pedro Alessandri 774, Santiago, Chile
E-mail: *kschiu@umce.cl*

Abstract In this paper, we investigate the existence, uniqueness and the asymptotic equivalence of a linear system and a perturbed system of differential equations with piecewise alternately advanced and retarded argument of generalized type (DEPCAG). This is based in the study of an equivalent integral equation with Cauchy and Green matrices type and in a solution of a DEPCAG integral inequality of Gronwall type. Several examples are also given to show the feasibility of results.

Key words piecewise constant argument of generalized type; hybrid equations; equivalence; asymptotic behavior; Gronwall's inequality

2010 MR Subject Classification 34A36; 34C41; 35M10; 26D10

1 Introduction

The problem of asymptotic equivalence between the solutions of two differential (or difference) equations shows an asymptotic relationship between two equations. If we know that two equations are asymptotically equivalent, and if we also know the asymptotic behavior of the solutions of one of them, then we can obtain information about the asymptotic behavior of the solutions of the other equation. Apparently, the first results concerning the asymptotic behaviour of systems on the basis of one-to-one correspondence between sets of solutions were obtained in [5, 15–17], see also [3, 4, 14, 22, 23].

Let $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ be the sets of all natural, real and complex numbers, respectively. Denote by $|\ .\ |$ the Euclidean norm in $\mathbb{C}^n$, $n \in \mathbb{N}$ and $\mathbb{R}^+ = [0, \infty)$. Fix two real valued non-negative sequences $\{t_i\}$, $\{\xi_i\}$, $i \in \mathbb{N}$ such that $t_i < t_{i+1}$, $t_i \le \xi_i \le t_{i+1}$ and $t_i \to \infty$ as $i \to \infty$. Let $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$, be a given general step function $\gamma|_{I_i} = \xi_i$, $I_i = [t_i, t_{i+1})$, $I_{-1} = [0, t_0]$, $\gamma(s) = s$ for s in I_{-1} and $\mathbb{R}^+ = \bigcup\limits_{i=-1}^{\infty} I_i$. In this case we speak of differential equations with

*Received December 4, 2016; revised January 8, 2017. This research was in part supported by FGI 05-16 DIUMCE.

piecewise constant arguments of generalized type, in short DEPCAGs. Theory and practice of the DEPCAGs were discussed extensively in [1, 2, 6–13, 18, 19].

To the best of our knowledge, there are only a few papers involving the DEPCAGs to investigate the problem of asymptotic equivalence between the solutions of two differential equations.

In 2008, Akhmet [2] proved the asymptotic equivalence of stable solutions of the following DEPCAGs

$$x'(t) = Cx(t), \tag{1.1}$$

$$z'(t) = Cz(t) + f(t, z(t), z(\gamma(t))), \tag{1.2}$$

where $x, z \in \mathbb{R}^n$, $t \in R$, C is a constant $n \times n$ real valued matrix and $f \in C(\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n)$ is a real valued $n \times 1$ function, $\gamma(t) = \xi_i$, $t \in [t_i, t_{i+1})$, if $i \in \mathbb{Z}$. Akhmet's stability result (our Corollary 4.5) for perturbed system (1.2) is obtained under stronger conditions than our ones. Akhmet had many difficulties, since he had not a global Gronwall-type lemma. See Remarks 2 and 3 in [6].

In 2009, Pinto [19] proved the asymptotic equivalence of stable solutions of the following DEPCAs

$$x'(t) = A(t)x(t),$$

$$z'(t) = A(t)z(t) + f(t, z(t), z(\gamma(t))), \tag{1.3}$$

where $x, z \in \mathbb{C}^n$, $n \times n$ complex valued matrices $A = A(t)$ is locally integrable in $\mathbb{R}^+$, $f : \mathbb{R}^+ \times \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}^n$ is a continuous function and $\gamma(t) = \xi_i$, $t \in [t_i, t_{i+1})$, if $i \in \mathbb{Z}$.

In the present paper, we consider the following linear differential equations

$$x'(t) = A(t)x(t), \tag{1.4}$$

$$y'(t) = A(t)y(t) + B(t)y(\gamma(t)), \tag{1.5}$$

and the linear perturbed DEPCAG system

$$z'(t) = A(t)z(t) + B(t)z(\gamma(t)) + f(t, z(t), z(\gamma(t))), \tag{1.6}$$

where $x, y, z \in \mathbb{C}^n$, the $n \times n$ complex valued matrices $A = A(t)$ and $B = B(t)$ are locally integrable in $\mathbb{R}^+$ and $f : \mathbb{R}^+ \times \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}^n$ is a continuous function.

We assume the following hypotheses.

(**L**) There exist two positive locally integrable functions η_1 and η_2 on $\mathbb{R}^+$ such that

$$|f(t, x_1, z_1) - f(t, x_2, z_2)| \le \eta_1(t)|x_1 - x_2| + \eta_2(t)|z_1 - z_2| \tag{1.7}$$

for all $t \in \mathbb{R}^+$, $x_1, x_2, z_1, z_2 \in \mathbb{C}^n$ and $f(t, 0, 0) = 0$.

(**M**) Let $\rho_i^+(A) = \exp(\int_{t_i}^{\gamma(t_i)} |A(s)|\mathrm{d}s)$, $\rho_i^-(A) = \exp(\int_{\gamma(t_i)}^{t_{i+1}} |A(s)|\mathrm{d}s)$, $\rho_i(A) = \rho_i^+(A)\rho_i^-(A)$ and $\nu_i^\pm(B) = \rho_i^\pm(A) \ln \rho_i^\pm(B)$.

Suppose

$$\rho(A) = \sup_{i \in \mathbb{N}} \rho_i(A) < \infty,$$

$$\nu_i^+(B) \le \nu^+ =: \sup_{i \in \mathbb{N}} \nu_i^+(B) < 1, \quad \nu_i^-(B) \le \nu^- =: \sup_{i \in \mathbb{N}} \nu_i^-(B) < 1.$$