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SOLVING COUPLED PSEUDO-PARABOLIC EQUATION USING A MODIFIED DOUBLE LAPLACE DECOMPOSITION METHOD*

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Abstract In this paper, the modification of double Laplace decomposition method is proposed for the analytical approximation solution of a coupled system of pseudo-parabolic equation with initial conditions. Some examples are given to support our presented method. In addition, we prove the convergence of double Laplace transform decomposition method applied to our problems.

Key words double Laplace transform; inverse double Laplace transform; singular parabolic equation; coupled pseudo-parabolic equation; single Laplace transform; decomposition methods

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1 Introduction

In theocratical physics, it is usually very important to seek and construct explicit solutions of linear and nonlinear partial differential equations (PDEs). Therefore, the solution helps the researchers to understand the physical phenomena. The parabolic equation occurs in several areas of applied mathematics, such as heat conduction, the phenomenea of turbulence and flow through a shock wave traveling in a viscous fluid such as the modeling of dynamics. In recent years, several studies for the coupled linear and nonlinear initial value problems arise in the literature. The pseudo-parabolic equation models a variety of physical processes. The one dimensional pseudo-parabolic equation was derived in [17]. In general, some of the nonlinear models of real-life problems are still very difficult to solve either theoretically or numerically. Recently, many authors have proposed analytical solution to one dimensional coupled parabolic equation (Burgers equation), eg. [7, 11] using Adomian decomposition method and [5] which proposed the homotopy perturbation method to obtain the exact solution of nonlinear Burgers' equation. In [3] the author used Laplace transform and homotopy perturbation method to obtain approximate solutions of homogeneous and inhomogeneous coupled Burgers' equation. The authors in [13, 18] obtained the approximate solution of the viscous coupled Burgers' equation using cubic and cubic B-spline collocation method. The convergence of Adomian's

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method was studied by several authors [1, 2, 4, 6]. In this work, modified double Laplace decomposition method and the self-canceling noise-terms phenomenon will be employed in the treatments of pseudo-parabolic equation. The main aim of this method is that it can be used directly without using restrictive assumptions or linearization. Now, we recall the following definitions which are given by [9, 14, 15]. The single Laplace transform of the function $f_1(x)$ and double Laplace transform of the functions $u(x,t)$ and $f(x,t)$ are defined as

$$L_x[f_1(x)] = F_1(p) = \int_0^\infty \mathrm{e}^{-px} f_1(x)\,\mathrm{d}x, \tag{1.1}$$

$$L_xL_t[u(x,t)] = U(p,s) = \int_0^\infty \int_0^\infty \mathrm{e}^{-px}\mathrm{e}^{-st} u(x,t)\mathrm{d}t\,\mathrm{d}x, \tag{1.2}$$

and

$$L_xL_t[f(x,t)] = F(p,s) = \int_0^\infty \int_0^\infty \mathrm{e}^{-px}\mathrm{e}^{-st} f(x,t)\mathrm{d}t\,\mathrm{d}x, \tag{1.3}$$

where $x,t > 0$ and p,s are complex values, and further double Laplace transform of the first order partial derivatives with respect to x and t are given by

$$L_xL_t\left[\frac{\partial u(x,t)}{\partial t}\right] = sU(p,s) - U(p,0). \tag{1.4}$$

Similarly, the double Laplace transform for the second partial derivative with respect to x and t are defined as follows

$$L_xL_t\left[\frac{\partial^2 u(x,t)}{\partial^2 t}\right] = s^2U(p,s) - sU(p,0) - \frac{\partial U(p,0)}{\partial t}. \tag{1.5}$$

The following basic lemma of the double Laplace transform is given and shall be used in this paper.

Lemma 1.1 Double Laplace transform of the non constant coefficient second order partial derivative $x\frac{\partial u}{\partial t}$ and the function $xf(x,t)$ are given by

$$L_xL_t\left(x\frac{\partial u}{\partial t}\right) = (-1)\frac{\mathrm{d}}{\mathrm{d}p}\left[sU(p,s) - U(p,0)\right], \tag{1.6}$$

and

$$L_xL_t\left(xf(x,t)\right) = (-1)\frac{\mathrm{d}}{\mathrm{d}p}\left[L_xL_t\left(f(x,t)\right)\right] = (-1)\frac{\mathrm{d}F(p,s)}{\mathrm{d}p}. \tag{1.7}$$

One can prove this lemma by the definition of double Laplace transform in Eq. (1.2), Eq. (1.4) and Eq. (1.5).

2 Singular One Dimensional Pseudo-Parabolic Equation

In this section we will use modified double Laplace decomposition method to solve singular one dimensional pseudo-parabolic equation. We consider singular one dimensional pseudo-parabolic with initial conditions in the form

$$\frac{\partial u}{\partial t} - \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial u}{\partial x}\right) - \frac{1}{x}\frac{\partial^2}{\partial x\partial t}\left(x\frac{\partial u}{\partial x}\right) = f(x,t), \tag{2.1}$$

subject to

$$u(x,0) = f_1(x), \tag{2.2}$$