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CASH SUBADDITIVE RISK MEASURES FOR PORTFOLIO VECTORS*

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Abstract In this paper, from the viewpoint of the time value of money, we study the risk measures for portfolio vectors with discount factor. Cash subadditive risk measures for portfolio vectors are proposed. Representation results are given by two different methods which are convex analysis and enlarging space. Especially, the method of convex analysis make the line of reasoning and the representation result be simpler. Meanwhile, spot and forward risk measures for portfolio vectors are also introduced, and the relationships between them are investigated.

Key words cash subadditivity; risk measures; convex analysis; portfolio vectors

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1 Introduction

In their seminal paper, Artzner et al. [2, 3] first introduced the concept of coherent risk measures by proposing four basic properties to be satisfied by every sound financial risk measure. Further Delbaen [11, 12] studied coherent risk measures on general probability spaces and risk measure for non-integrable random variables. Chen and Ye [9] introduced $\mathcal{F}_\tau$-coherent risk measures and obtained its representation result. Föllmer and Schied [17–19] and independently, Frittelli and Rosazza Gianin [20] introduced the broader class, named convex risk measures by dropping one of the coherency axioms (see [21, 32]). The convex risk measures were also studied by Deprez and Gerber [14] in the context of insurance. Over the last decade, the above axiomatic approach to risk measures have been largely accepted by academics and practitioners. In this axiomatic approach, the axiom of cash additivity (also called translation invariance) is employed. Krätschmer [25, 26] gave the representation of convex risk measures for unbounded financial position in the presence of uncertainty about the market model by convex analysis.

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Karoui and Ravanelli [15] introduced the concept of cash subadditive risk measures and provided the representation results by enlarging space.

Recently, multivariate scalar risk measures have been studied (see [4, 5–7, 13, 16, 22–24, 27, 28, 30, 31]). Burgert and Rüschendorf [6] first introduced multivariate (also called multidimensional) scalar coherent and convex risk measures (in their terminology, risk measures for portfolio vectors was used), and provided the representation results. Wei and Hu [31] further extended the framework of Burgert and Rüschendorf [5] to more general settings, and provided the representation results for scalar coherent and convex risk measures for portfolio vectors.

In the study of scalar risk measures for portfolio vectors, the axiom of cash additivity is also employed. However, like the case of the univariate risk measures, the cash additivity is not always the case for multivariate risk measures (see [1, 8, 10, 15]). In the present paper, we study the dual representation of cash subadditive risk measures for portfolio vectors. To obtain duality results we can utilize two approaches, one is convex analysis tool, the other is the method of enlarging the space of risky positions. For convex analysis, it is the most important to get the essential properties of risk measures which are monotonicity and convexity. Consequently, the dual representations which is obtained by convex analysis is simpler and more significant. on the other hand, the representation result is described in terms of the enlargement which makes it unnecessarily complicated. However, the approach of enlarging the space is more richer financial interpretation, for instance, it provides an interesting interpretation of cash additive and cash subadditive risk measures where default events or stochastic numéraires are taken into account. The new risk measures for portfolio vectors are characterized by penalty function defined on a set of product sublinear probability measure and can be represented using penalty function associated with cash additive risk measures for portfolio vectors on some extend spaces. By taking into account the time value of money, spot and forward multivariate risk measures are also introduced and the relationships between them are also investigated. Finally, the expressions of multivariate cash subadditive risk measures in terms of spot and forward multivariate risk measures are given.

The rest of the paper is organized as follows. In Section 2, we briefly state preliminaries. In Section 3, we will introduce the concepts of spot and forward risk measures for portfolio vectors, and their properties will also be discussed. In Section 4, we will introduce the cash subadditive risk measures for portfolio vectors and provide the representation results by different methods which are convex analysis and enlarging space. Finally, conclusions are summarized in Section 5.

2 Preliminaries

In this section we introduce some notations and we recall some key theorems of risk measures for portfolio vectors. The following definitions and theorems are from Krätschmer [26], Karoui and Ravanelli [15], Rüschendorf [29] and Wei and Hu [31].

2.1 Risk Measures

Let $(\Omega, \mathcal{F})$ be a fixed measurable space and $(\Omega, \mathcal{F}, P)$ be a fixed probability space. We denote $\mathcal{G} = L^\infty(\Omega, \mathcal{F})$ and $\mathcal{H} = L^p(\Omega, \mathcal{F}, P)$ for $1 \le p \le \infty$, where, when $1 \le p < \infty$, $\mathcal{H}$ denotes the space of random variables with p-order moment. For any $X, Y \in \mathcal{H}$, we will identify X