Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2017,**37B**(6):1594–1606
Acta Mathematica Scientia 数学物理学报
http://actams.wipm.ac.cn

DETERMINING THE DISCRIMINATING DOMAIN FOR HYBRID LINEAR DIFFERENTIAL GAME WITH TWO PLAYERS AND TWO TARGETS*

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Abstract This paper studies a bounded discriminating domain for hybrid linear differential game with two players and two targets using viability theory. First of all, we prove that the convex hull of a closed set is also a discriminating domain if the set is a discriminating domain. Secondly, in order to determine that a bounded polyhedron is a discriminating domain, we give a result that it only needs to verify that the extreme points of the polyhedron meet the viability conditions. The difference between our result and the existing ones is that our result just needs to verify the finite points (extreme points) and the existing ones need to verify all points in the bounded polyhedron.

Key words viability; hybrid linear differential game; discriminating domain; nonsmooth analysis

2010 MR Subject Classification 91A23; 91A25

1 Introduction

Hybrid control systems typically arise from computer-aided control, manufacturing, communication networks, autopilot design, traffic management systems, industrial process control and the hierarchical organization of complex control systems [1]. They contain two subsystems that one with continuous dynamics and the other with discrete-event dynamics [2–4]. They interact with each other. In [5], the necessary conditions of a class of optimal control problem for the singularly perturbed hybrid dynamical systems is obtained. In [6], the existence of solutions for a nonlocal hybrid boundary value problem of Caputo fractional integro-differential equations is discussed. The investigation of hybrid systems is creating a new bridge among control engineering, mathematics and computer science.

*Received January 5, 2017; revised March 1, 2017. This work was supported by National Science Foundation of China (11171221), Doctoral Program Foundation of Institutions of Higher Education of China (20123120110004), Natural Science Foundation of Shanghai (14ZR1429200), Innovation Program of Shanghai Municipal Education Commission (15ZZ073) and Key Research Project Plan of Institutions of Higher of Henan Province (17A120010).

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Differential game usually consists of two players which are the pursuer and the evader, with conflicting goals. They are a part of control theory with conflicts between the players. The dynamics of the pursuer and the evader are described by differential equations. During the evolution, two players are allowed to control the differential equation by inputs of his opponent, respectively. Each player attempts to control the state of the system so as to achieve his goal, the system responds to the inputs of both players. The systematic research of differential game was initiated by Isaacs in the 1950s. Although differential game is related closely with optimal control problems, there is a little difference between the two. The difference is that there is a single control input $u(t)$ and a single criterion to be optimized in an optimal control problem and differential game theory generalizes this to two control inputs $u(t), v(t)$ and two criteria.

Hybrid differential game is a differential game of two players on a hybrid system where the players have control over some discrete time choices during the evolution of the system and the continuous dynamics stays deterministic. They can model discrete aspects like decision delays and differential game with different controls in different modes of the system.

Based on viability theory, the study for differential game started with the pioneering works of Aubin [7] and for hybrid differential game in [8, 9]. Krasovskii and Subbotin designed optimal feedbacks of the players of differential game in [10]. Leong and Huang studied the stochastic differential game of capitalism in [11]. Cardallaguet studied differential game with one target, two players and characterizes the victory domain as the largest set satisfying some geometric conditions in [12]. Cardallaguet, Quincampoix and Saint-Pierre gave numerical method of victory domain in [13]. In [14], a method to determine the viability of linear differential inclusions on an unbounded polyhedron was proposed. The theory was generalized to hybrid systems in [15]. Multiple linear model in [16, 17] was largely used in identification and control applications. In [16], each multiple linear model was a local representation of the hybrid system. These models were combined with Bayes theorem to describe the nonlinear hybrid system. In [17], the authors presented a systematic multiple model approach based on multiple linear model to describe nonlinear switched systems. In [18], a two player zero sum differential game with infinitely many initial positions and without Isaacs condition was proposed. Using optimal transportation theory and stochastic control, there exists a value of the game with such random strategies.

In this paper, using viability theory, a bounded discriminating domain for hybrid linear differential game with two players and two targets is studied. Based on continuous linear differential game, we give two results about hybrid linear differential game. One is that the convex hull of a discriminating domain is also a discriminating domain and that the discriminating kernel is a convex set. The other is that a bounded polyhedron is expressed as the convex hull of it's finite extremes points. We prove that the bounded polyhedron is a discriminating domain if the extremes points of the polyhedron meet the viability conditions. Our results simplify the existing ones. Then, we give an example to demonstrate the results.

The rest of the paper is organized as follows. Section 2 reviews some basic notions and properties of hybrid systems and game theory. In Section 3, it is discussed that the convex hull of a discriminating domain is also a discriminating domain for hybrid linear differential game. Then, it is researched that a bounded polyhedron is a discriminating domain.