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A NEW PERTURBATION THEOREM FOR MOORE-PENROSE METRIC GENERALIZED INVERSE OF BOUNDED LINEAR OPERATORS IN BANACH SPACES*

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Abstract In this paper, we investigate a new perturbation theorem for the Moore-Penrose metric generalized inverses of a bounded linear operator in Banach space. The main tool in this paper is "the generalized Neumann lemma" which is quite different from the method in [12] where "the generalized Banach lemma" was used. By the method of the perturbation analysis of bounded linear operators, we obtain an explicit perturbation theorem and three inequalities about error estimates for the Moore-Penrose metric generalized inverse of bounded linear operator under the generalized Neumann lemma and the concept of stable perturbations in Banach spaces.

Key words Banach space; bounded linear operator; metric projection; generalized inverse; perturbation analysis; Moore-Penrose; quasi-additive

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1 Introduction

It is well known that perturbation analysis for the linear generalized inverse of bounded linear operators in Banach spaces is very important in applications for lots of mathmatical branches such as frame theory (see [1, 2]), approximation theory (see [3, 6]), control theory, computation and nonlinear analysis (see [4, 5]). In recent years, the perturbation study of generalized inverses with the help of the concepts of the local fine point [4], the stable perturbation [7] and the gap between closed subspaces [8] has appeared in many works (see [3, 4, 6, 9–13]). For instance, in [3], Nashed and Chen showed that the generalized inverse T^+ is stable in the class of the perturbations satisfying the following conditions:

$$\left[I+\left(\overline{T}-T\right)T^{+}\right]^{-1}\overline{T}N\left(T\right) \quad\subset\quad R\left(T\right).$$

In 2008, Jipu Ma, a student of Tseng, improved the above results of Nashed and Chen. Following from this result, a generalized transversality theorem in global analysis is obtained (see [4]).

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Let X and Y be two Banach spaces. Let $L(X,Y), C(X,Y)$ and $B(X,Y)$ denote the linear space of all linear operators, the homogeneous set of all closed linear operators with a dense domain and the Banach space of all bounded linear operators from X into Y, respectively. For any $T \in L(X,Y)$, we denote by $D(T), N(T)$ and $R(T)$, the domain, the null space and the range of T, respectively. For an operator $T \in C(X,Y)$ with domain $D(T)$ is dense in X, but not identical with the whole space X, T must be unbounded. The situation is quite different and one may encounter various difficulties. We assume that $T \in C(X,Y)$ with a closed range $R(T)$, X is the topological direct sum of the null space $N(T)$ and a closed subspace R^+ of X, Y is the topological direct sum of the range $R(T)$ and a closed subspace N^+ of Y, i.e., $X = N(T) \oplus R^+$ and $Y = R(T) \oplus N^+$. Let P be the projector of X onto $N(T)$ along R^+ and Q be the projector of Y onto $R(T)$ along N^+. An operator $T^+ \in B(Y,X)$ is called an oblique projection generalized inverse of T with respect to P,Q, if (1) $TT^+T = T$ on $D(T)$; (2) $T^+TT^+ = T^+$ on Y; (3) $T^+T = I - P$ on $D(T)$; (4) $TT^+ = Q$ on Y (see [25]).

In 2008, in [14], Wang and Zhang showed a perturbation theorem for the oblique projection generalized inverse under conditions that δT is T-bounded, $a\,||T^+|| + b\,||Q|| < 1$ and $N(T) \subset N(\delta T)$.

More interestingly, in 2003 [17], Ding established new perturbation results on pseudo-inverse of linear operators in Banach spaces, the method used is quite different from all the methods which were used before. So motivated by the idea in [17], in 2010 (see [18]), Yang and Wang gave some new perturbation theorems for generalized inverses of linear operators in Banach spaces whose the main tool is "the generalized Neumann lemma" which is quite different from that in [14].

In 2011, in [15], Huang improved the above results of Wang and Zhang for the oblige projection generalized inverse under conditions that δT is T-bounded with $b < 1$ and gave another form of perturbation theorem.

However, linear generalized inverses are not appropriate for establishing the extremal solutions, the minimal norm solutions and the best approximation solutions of an ill-posed linear operator equations in Banach spaces [20]. In order to solve the best approximation problem for ill-posed linear operator equations in Banach spaces, it is necessary to study the metric generalized inverses of linear operators between Banach spaces. In 1974, in [20], Nashed and Votruba first introduced the concept of metric generalized inverse.

In 2001, in [26], Wang and Yu gave the character and representive of class of metric projection in Banach space.

In 2003, in [16], Wang and Wang gave the definition of the Moore-Penrose metric generalized inverse of a linear operator (see Definition 2.5).

Since T^M is a homogenous, nonlinear operator in general Banach space, so the perturbation analysis for the Moore-Penrose metric generalized inverse of a bounded linear operator in Banach space is quite different from that of the linear generalized inverse.

In 2014, in [12], Ma and Wang et al. showed a perturbation theorem for Moore-Penrose metric generalized inverse $\overline{T}^M$, $\overline{T} = T + \delta T$ under conditions that $T \in B(X,Y)$ and $\delta T \in B(X,Y)$ with $||T^M|| \cdot ||\delta T|| < 1$, $N(T) \subset N(\delta T)$, $R(\delta T) \subset R(T)$, they obtained the following results.