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A VARIATIONAL-HEMIVARIATIONAL INEQUALITY IN CONTACT PROBLEM FOR LOCKING MATERIALS AND NONMONOTONE SLIP DEPENDENT FRICTION*

Stanisław MIGÓRSKI
Chair of Optimization and Control, Jagiellonian University in Kraków, ul. Łojasiewicza 6, 30348 Kraków, Poland
E-mail: *stanislaw.migorski@uj.edu.pl*

Justyna OGORZAŁY
Faculty of Mathematics and Computer Science, Jagiellonian University in Kraków, ul. Łojasiewicza 6, 30348 Kraków, Poland
E-mail: *justyna.ogorzaly@gmail.com*

Abstract We study a new class of elliptic variational-hemivariational inequalities arising in the modelling of contact problems for elastic ideally locking materials. The contact is described by the Signorini unilateral contact condition and the friction is modelled by the nonmonotone multivalued subdifferential condition which depends on the slip. The problem is governed by a nonlinear elasticity operator, the subdifferential of the indicator function of a convex set which describes the locking constraints and a nonconvex locally Lipschitz friction potential. The result on existence and uniqueness of solution to the inequality is shown. The proof is based on a surjectivity result for maximal monotone and pseudomonotone operators combined with the application of the Banach contraction principle.

Key words variational-hemivariational inequality; Clarke subdifferential; locking material; unilateral constraint; nonmonotone friction

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1 Introduction

In this paper we study a class of contact problems for elastic ideally locking materials. The contact is assumed to be static and it is described by the Signorini unilaterial contact condition with a nonmonotone friction condition between a locking body and a rigid foundation. The constitutive law is modelled by the (convex) subdifferential of the indicator function of a convex set which characterizes the locking constraints. On the other hand, the friction is described by

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the (Clarke) subdifferential boundary condition involving a locally Lipschitz function which, in general, is nonsmooth, nonconvex and nondifferentiable. Moreover, the multivalued frictional contact depends on the slip which is important in many applications. In consequence, the convex analysis approach to such problem is not adequate and the weak formulation has a form of variational-hemivariational inequality of elliptic type.

Variational-hemivariational inequalities represent a special class of inequalities, in which both convex and nonconvex functions are involved. Very recently, they were studied in several papers (cf. [7, 11, 12]) in the context of modelling of various problems of Contact Mechanics. In [7] Han et al. studied existence and uniqueness of a solution for static variational-hemivariational inequalities and they showed continuous dependence of the solution on the data. Using the numerical approximation they proved a convergence result and derived error estimates. Finally, they applied abstract results in the analysis of static contact problem with friction for elastic materials. In [11], the authors dealt with a viscoelastic problem on infinite time interval in which the contact is frictionless and is modeled with a boundary condition which describes both the instantaneous and the memory effects of the foundation. They proved that this problem leads to a variational-hemivariational inequality with the history-dependent operators. In turn, in [12] Migórski et al. considered a class of elliptic variational-hemivariational inequalities in reflexive Banach spaces. They stated and proved an existence and uniqueness result and two convergence results, one of them concerns the continuous dependence of the solution with respect to the data, while the second one is obtained by means of penalization method. Using abstract results, they studied an elastic contact problem with unilateral constraint.

In the present paper, we use variational-hemivariational inequalities to investigate a new static contact problem for elastic ideally locking materials. To the best of our knowledge, this is the first work on variational-hemivariational inequalities for locking materials. The theory of locking materials was initiated by Prager [16–18]. The variational problems encountered in the theory of locking materials were studied in [2], where the equivalence between the statical and the kinematical methods for such materials were shown.

Locking materials belong to a class of hyperelastic bodies for which the strain tensor is constrained to stay in a convex set. Let B be a closed, convex subset of $\mathbb{S}^d$ with $\mathbf{0}_{\mathbb{S}^d} \in B$, where $\mathbb{S}^d$ stands for the space of second order symmetric tensors on $\mathbb{R}^d$. The elastic ideally locking materials are characterized by the following law

$$\begin{cases} \sigma_{ij} = \sigma_{ij}^e + \sigma_{ij}^l, \quad \sigma_{ij}^e = a_{ijkl}\varepsilon_{kl}(\boldsymbol{u}), \\ \boldsymbol{\varepsilon}(\boldsymbol{u}) \in B, \quad \sigma_{ij}^l \cdot (\varepsilon_{ij}^* - \varepsilon_{ij}(\boldsymbol{u})) \le 0 \ \text{ for all } \ \boldsymbol{\varepsilon}^* = (\varepsilon_{ij}) \in B. \end{cases} \tag{1.1}$$

Here σ_{ij}^e and σ_{ij}^l are elastic and locking components of the stress tensor σ_{ij} and $\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\varepsilon_{ij}(\boldsymbol{u}))$ is the infinitesimal strain tensor defined by $\varepsilon_{ij}(\boldsymbol{u}) = \frac{1}{2}(u_{i,j} + u_{j,i})$. The indices i, j, k, l run between 1 and d, and the summation convention over repeated indices is used.

In the one dimensional case, the typical strain-stress law of the type (1.1) is of the form

$$\sigma = \begin{cases} 0, & \text{if } \ \varepsilon < 0, \\ a\varepsilon, & \text{if } \ 0 \le \varepsilon < \varepsilon_0, \\ [a\varepsilon_0, +\infty), & \text{if } \ \varepsilon = \varepsilon_0, \\ \emptyset, & \text{if } \ \varepsilon > \varepsilon_0 \end{cases}$$