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SOLITARY WAVE AND PERIODIC WAVE SOLUTIONS FOR THE NON-NEWTONIAN FILTRATION EQUATIONS WITH NONLINEAR SOURCES AND A TIME-VARYING DELAY*

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Abstract This paper is concerned with the non-Newtonian filtration equations with nonlinear sources and a time-varying delay. By an extension of Mawhin's continuation theorem and some analysis methods, several sufficient conditions ensuring the existence of solitary wave and periodic wave solutions are obtained. Some corresponding results in the literature are improved and extended. An example is given to illustrate the effectiveness of our results.

Key words solitary wave; periodic wave; Mawhin's continuation theorem; time-varying delay; nonlinear sources

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1 Introduction

In this paper, we consider the solitary wave and periodic wave solutions for the following non-Newtonian filtration equation

$$\frac{\partial q}{\partial t}=\frac{\partial}{\partial x}\left(\left|\frac{\partial q}{\partial x}\right|^{p-2}\frac{\partial q}{\partial x}\right)+f(q_{\delta(t)})+g(t,x),\ t\geq 0,\ x\in\mathbb{R}, \tag{1.1}$$

where $p>1$, $f\in C(\mathbb{R},\mathbb{R})$, $g\in C(\mathbb{R}\times\mathbb{R},\mathbb{R})$, $q_{\delta(t)}(t,x)=q(t-\delta(t),x)$ and $\delta\in C(\mathbb{R},\mathbb{R})$.

In the last four decades, the evolutionary p-Laplacian, which comes from modeling fluid dynamics, has been studied widely. For related paper, we refer the reader to the papers [2, 4, 5, 15–18, 20–24] and the references therein.

The particular feature of evolutionary p-Laplacian is its gradient-dependent diffusivity. Such equations, and their stationary counterparts, appear in different models in non-Newtonian fluids, turbulent flows in porous media, certain diffusion or heat transfer processes, and recently in image processing. Suppose a compressible fluid flows in a homogeneous isotropic rigid porous

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medium. Then the volumetric moisture content θ, the seepage velocity $\overrightarrow{V}$ and the density of the fluid are governed by the continuity equation

$$\theta\frac{\partial \rho}{\partial t}+\operatorname{div}\big(\rho\overrightarrow{V}\big)=0.$$

For non-Newtonian fluid, the linear Darcy's law is no longer valid, because of the influence of many factors such as the molecular and ion effects. Instead, one has the following nonlinear relation:

$$\rho\overrightarrow{V}=-\lambda|\nabla P|^{\alpha-1}\nabla P,$$

where $\rho\overrightarrow{V}$ and $P=c\rho$ denote the momentum velocity and pressure respectively, $\lambda>0$ and $\alpha>0$ are some physical constants. After changing variables and notations, the non-Newtonian filtration equation is derived.

Recently, in [14], Liang et al. considered the following non-Newtonian filtration equation with nonlinear sources:

$$\frac{\partial q}{\partial t}=\frac{\partial}{\partial x}\left(\left|\frac{\partial q}{\partial x}\right|^{p-2}\frac{\partial q}{\partial x}\right)+f(q)+g(t,x),$$

where $p>1$, $f\in C(\mathbb{R},\mathbb{R})$ and $g\in C(\mathbb{R}\times\mathbb{R},\mathbb{R})$. By an extension of Mawhin's continuation theorem and some analysis methods, the authors obtained the existence results of solitary wave and periodic wave solutions.

Differential equations with time delays are traditionally formulated for spatially homogeneous equations, and different kinds of delayed equations are proposed by many authors, see for example [1, 3]. It is worthwhile to study the non-Newtonian filtration equation with a time-varying delay. However, to the best of our knowledge, the study on the existence of solitary wave and periodic wave solutions for the non-Newtonian filtration equations with a time-varying delay is a new topic, which seems not to have been considered in the literature.

Motivated by the above fact, the aim of this paper is to discuss the existence of solitary wave and periodic wave solutions for (1.1). Throughout this paper, we assume that there exists $e\in C(\mathbb{R},\mathbb{R})$ such that $g(t,x)=e(x+ct)$, where $c\in\mathbb{R}$. Let $q(t,x)=u(s)$ with $s=x+ct$ be the solution of Eq. (1.1), and let $\tau(s)=c\delta(t)$. Then Eq. (1.1) is transformed into the following form

$$cu'(s)=\big(\varphi_p(u'(s))\big)'+f(u(s-\tau(s)))+e(s),\quad s\in\mathbb{R},\tag{1.2}$$

where $\varphi_p(u'(s))=|u'(s)|^{p-2}u'(s)$.

Remark 1.1 In the case of $\delta(t)\equiv 0$, equation (1.1) will degenerate into the problem studied in [14], which implies that problem studied in [14] is the special case of (1.1). So we extend the results in [14] to the delay case.

Definition 1.2 Suppose that $u(s)$ is a solution of system Eq. (1.2) for $s\in\mathbb{R}$, $u(s)$ is called a solitary wave solution if $\lim\limits_{s\to-\infty}u(s)=\lim\limits_{s\to+\infty}u(s)$. Usually, a solitary wave solution of Eq. (1.1) corresponds to a homoclinic solution of Eq. (1.2). Suppose that $u(s)$ is a solution of system Eq. (1.2) for $s\in\mathbb{R}$, $u(s)$ is called a periodic wave solution if $u(s+T)\equiv u(s)$, where $T>0$ is a constant. Usually, a periodic wave solution of Eq. (1.1) corresponds to a periodic solution of Eq. (1.2).