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SMOOTHING NEWTON ALGORITHM FOR THE CIRCULAR CONE PROGRAMMING WITH A NONMONOTONE LINE SEARCH*

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Abstract In this paper, we present a nonmonotone smoothing Newton algorithm for solving the circular cone programming (CCP) problem in which a linear function is minimized or maximized over the intersection of an affine space with the circular cone. Based on the relationship between the circular cone and the second-order cone (SOC), we reformulate the CCP problem as the second-order cone problem (SOCP). By extending the nonmonotone line search for unconstrained optimization to the CCP, a nonmonotone smoothing Newton method is proposed for solving the CCP. Under suitable assumptions, the proposed algorithm is shown to be globally and locally quadratically convergent. Some preliminary numerical results indicate the effectiveness of the proposed algorithm for solving the CCP.

Key words circular cone programming; second-order cone programming; nonmonotone line search; smoothing Newton method; local quadratic convergence

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1 Introduction

Circular cone programming (CCP) problems (see [1, 2]) are an important class of convex optimization problems in which we minimize or maximize the linear function over the intersection of an affine space with the circular cone. In this paper, we focus on the following CCP problem

$$(P)\ \min\ \left\{c^T x : Ax = b,\ x \in C_\theta^n\right\}, \tag{1.1}$$

where $\theta \in (0, \frac{\pi}{2})$ is a given angle, $A \in R^{m\times n}$, $c \in R^n$ and $b \in R^m$ are the data, $x \in C_\theta^n$ is the variable. And the set C_θ^n is the n-dimension circular cone (CC), which is expressed as

$$C_\theta^n := \left\{x = (x_1, x_{2:n}) \in R \times R^{n-1} :\ \cos\theta\|x\| \le x_1\right\}, \tag{1.2}$$

where $x_{2:n} = (x_2, \cdots, x_n) \in R^{n-1}$, and $\|\cdot\|$ refers to the Euclidean norm of a vector.

As a special kind of the non-self-dual cone, the circular cone [3] is a pointed closed convex cone having hyperspherical sections orthogonal to its axis of revolution around which the cone is invariant to rotation. When the rotation angle $\theta = 45°$, the n-dimension circular cone reduces to the n-dimension second-order cone (SOC) K^n given by

$$K^n := \left\{x = (x_1, x_{2:n}) \in R \times R^{n-1} : \|x_{2:n}\| \le x_1\right\}. \tag{1.3}$$

Thus, the CCP includes the second-order cone programming (SOCP) [4] as a special case. The dual problem of (1.1) is

$$(D)\ \max\ \left\{b^T y : A^T y + s = c,\ s \in (C_\theta^n)^*\right\}, \tag{1.4}$$

where $y \in R^m$ is the variable, and $s \in (C_\theta^n)^*$ is the slack variable, here $(C_\theta^n)^*$ is the dual cone of C_θ^n defined by

$$(C_\theta^n)^* := \left\{x = (x_1, x_{2:n}) \in R \times R^{n-1} : \|x_{2:n}\| \le x_1 \cot\theta\right\}.$$

The sets of strictly feasible solutions of (1.1) and (1.4) are

$$F^0(P) = \{x : Ax = b,\ x \in \mathrm{int}C_\theta^n\},$$

$$F^0(D) = \left\{(y, s) : A^T y + s = c,\ s \in \mathrm{int}(C_\theta^n)^*\right\},$$

respectively, where $\mathrm{int}C_\theta^n$ (respectively, $\mathrm{int}(C_\theta^n)^*$) denotes the interior of C_θ^n (respectively, $(C_\theta^n)^*$). Throughout this paper, we assume that both (1.1) and (1.4) are strictly feasible, i.e., $F^0(P) \times F^0(D) \ne \emptyset$. Thus, it can be shown that both (1.1) and (1.4) have optimal solutions, and finding optimal solutions of the CCP (1.1) and (1.4) is equivalent to solving the following optimality conditions, i.e., KKT conditions,

$$\begin{cases} Ax = b,\ x \in C_\theta^n, \\ A^T y + s = c,\ s \in (C_\theta^n)^*, \\ x^T s = 0. \end{cases} \tag{1.5}$$

Unfortunately, because $(C_\theta^n)^*$ and C_θ^n in the above conditions are not usually the same cone with $\theta \ne 45°$, we have a formidable task to directly apply smoothing Newton algorithms