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GLOBALLY ATTRACTING SOLUTIONS TO IMPULSIVE FRACTIONAL DIFFERENTIAL INCLUSIONS OF SOBOLEV TYPE*

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Abstract We study a generalized Cauchy problem associated with a class of impulsive fractional differential inclusions of Sobolev type in Banach spaces. Our aim is to prove the existence of a compact set of globally attracting solutions to the problem in question. An application to fractional partial differential equations subject to impulsive effects is given to illustrate our results.

Key words globally attracting solution; impulsive condition; nonlocal condition; condensing map; measure of non-compactness; MNC-estimate

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1 Introduction

We are concerned with the following problem in a Banach space X

$$D_0^\alpha Bu(t) \in Au(t) + F(t, u(t)),\ t \neq t_k, t_k \in (0, +\infty), k \in \Lambda, \tag{1.1}$$

$$\Delta u(t_k) = I_k(u(t_k)), \tag{1.2}$$

$$u(0) = g(u), \tag{1.3}$$

where $D_0^\alpha, \alpha \in (0, 1)$, is the fractional derivative in the Caputo sense, A and B are linear, closed and unbounded operators in X, $\Lambda \subset \mathbb{N}$, $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$. The functions F, g and I_k will be specified in Section 3.

The study of the Sobolev type equations can be traced back to the work of Barenblat et al. [5], in which the authors initiated a model of flow of liquid in fissured rocks, i.e., the equation

$$\partial_t(u - \partial_x^2 u) - \partial_x^2 u = 0.$$

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This model then was developed and studied in [7, 25] when the authors considered the abstract nonlinear equation

$$\frac{\mathrm{d}}{\mathrm{d}t}Bu(t) - Au(t) = f(t, u(t))$$

in Banach spaces, where A and B are unbounded operators.

Recently, as the fractional calculus becomes a powerful tool for describing various physical phenomena such as flows in porous media, oscillations and controls (see, e.g. [17, 23, 26]), fractional differential equations were considered as an alternative tool in modeling. As a matter of fact, the fractional differential equations of Sobolev type attracted many researches in the last few years. We refer the reader to [3, 4, 15, 18, 24] for some recent results on solvability and controllability which are close to our work.

As far as the system (1.1)–(1.3) is concerned, the appearance of multi-valued nonlinearity F is motivated by a number of problems: differential equations (DEs) with discontinuous right-hand side (see [16]), differential variational inequalities (see [28]), feedback controls (see [20]), etc. Regarding the impulsive condition in (1.2), this is an effect appeared as the state function stands abrupt changes, which happen frequently in biology and engineering. The non-local condition in (1.3) was first studied in [10] and considered as a better description for initial condition than that in classical Cauchy problem. In applications, the non-local condition is usually in the following forms

$$u(0) = u_0 + \sum_{i=1}^{m} c_i u(t_i), c_i \in \mathbb{R}, t_i > 0,$$

$$u(0) = u_0 + \frac{1}{b}\int_0^b k(s)u(s)\mathrm{d}s, b > 0, k \text{ is a real function.}$$

It should be mentioned that, impulsive fractional differential equations (IFrDEs) were an attractive subject in recent years. Concerning IFrDEs in finite dimensional spaces with initial/boundary conditions, we refer to [33] for solvability and stability of Ulam type results. For a complete reference for studies in this direction, see [30, 32]. In addition, apart from IFrDEs with Caputo derivative, a formulation and existence of solutions for IFrDEs involving Hadamard derivative can be found in [34]. Referring to semilinear IFrDEs in Banach spaces, the authors in [29] gave an explicit way to represent mild solutions. By using this formulation and fixed point approach, a number of existence results was obtained (see e.g. [29–31]).

An important question associated with the problem (1.1)–(1.3) is to address the large-time behavior of its solutions. It should be noted that the theory of global attractors (see, e.g. [11]) does not work in this case due to the lack of semigroup property of solution operator. In addition, the using of Lyapunov function to analyze stability of solutions is impractical due to the difficulty in computing and estimating fractional derivatives, even in finite dimensional case. By this reason, results on large-time behavior of solutions to IFrDEs were little known in literature. In the recent papers [12, 21, 22], we studied some models of semilinear fractional DEs in Banach spaces involving non-local conditions and impulsive effects, in which the existence of attracting solutions was proved by employing the contraction mapping principle. This approach was introduced by Burton and Furumochi [8, 9] in dealing with stability for ordinary/functional differential equations. However, the techniques used in [12, 21, 22] do not work for our problem in this note since the nonlinear functions F, g and I_k are not Lipschitzian in our settings (see