Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2017,**37B**(5):1331–1347
Mathematica Acta Scientia 数学物理学报
http://actams.wipm.ac.cn

LOCAL AND PARALLEL FINITE ELEMENT METHOD FOR THE MIXED NAVIER-STOKES/DARCY MODEL WITH BEAVERS-JOSEPH INTERFACE CONDITIONS*

Guangzhi DU (杜光芝)†
School of Mathematics and Statistics, Shandong Normal University, Jinan 250014, China
E-mail: *guangzhidu@gmail.com*

Liyun ZUO (左立云)
School of Mathematical Sciences, University of Jinan, Jinan 250022, China
E-mail: *yeziliyun@126.com*

Abstract In this paper, we consider the mixed Navier-Stokes/Darcy model with Beavers-Joseph interface conditions. Based on two-grid discretizations, a local and parallel finite element algorithm for this mixed model is proposed and analyzed. Optimal errors are obtained and numerical experiments are presented to show the efficiency and effectiveness of the local and parallel finite element algorithm.

Key words Navier-Stokes equations; Darcy's law; two-grid algorithm; Beavers-Joseph interface conditions; parallel finite element method

2010 MR Subject Classification 35M30; 65N12; 65N30

1 Introduction

Computational modeling of coupled Navier-Stokes and Darcy flows has aroused great attention due to its wide applications, such as, the interaction between surface and subsurface flows, blood motion in the vessels, industrial filtrations, and so on (see [1, 3–6, 9–12, 15, 32] and the references cited therein). The fluid flow is modeled by the Navier-Stokes equations, porous media flow is governed by the Darcy equations, two flows are coupled through certain interface conditions.

A lot of works can be found for the coupled Stokes (or Navier-Stokes) and Darcy problem with the Beavers-Joseph-Saffman interface conditions, see e.g., [6–8, 11–14, 16–18, 22, 23] and the references cited therein. The main reason is that the well-posedness of the coupled problem with Beavers-Joseph interface conditions were not demonstrated. However, the Beavers-Joseph-Saffman condition ignores certain contributions made by the flow in the porous media flow to

*Received April 6, 2016; revised November 9, 2016. Supported by NSFC (11571274 and 11401466).
†Corresponding author: Guangzhi DU.

the coupling of the two models; the ignored contributions may be very important in some applications such as karst aquifers. In [5], Chen et al. demonstrated that the Beavers-Joseph interface boundary condition is more accurate than the Beavers-Joseph-Saffman interface boundary condition or its further simplifications.

A mathematical difficulty arises from the fact that one need to couple two different partial differential equations. The decoupling approach therefore becomes one popular approach to solving this mixed model. It allows one to tailor algorithm components flexibly and exploit the existing computing resources effectively for each local model. Mu and Xu successfully applied the two-grid method to solve the mixed Stokes-Darcy problem in [12]. Subsequently, in [16] Cai and coworkers used two-grid method to decouple and linearize the mixed Naver-Stokes/Darcy problem. Recently, a modified two-grid method was proposed for the steady Stokes-Darcy model with the Beavers-Joseph-Saffman interface condition in [20, 22] and with the Beavers-Joseph interface condition in [19]. In [21], the author presented the existence and local uniqueness of a weak solution for the Navier-Stoke/Darcy problem with Beavers-Joseph interface boundary conditions.

Using above two-grid methods, we note that on the fine grid the linearized Navier-Stokes subproblem takes much more time than the Darcy subproblem. To further improve the effectiveness of solving the mixed Navier-Stokes/Darcy problem, in this paper, we propose a local and parallel algorithm to solve the mixed Navier-Stokes/Darcy problem. The idea is similar to [22–26, 28–31], the low frequency components can be approximated well by a relatively coarse grid and high frequency components can be computed on a fine grid by some local and parallel procedure.

In our method, on the coarse grid, we solve a coupled Navier-Stokes/Darcy problem; on the fine grid, we first solve a series of independent local discrete Darcy subproblems with a coarse grid approximation u_H to the interface coupling conditions, then solve a series of independent local linear Navier-Stokes subproblems with the numerical solution of Darcy problem on fine grid to the interface coupling conditions in parallel. The numerical tests show our scheme is efficient and effective.

The rest of the paper is organized as follows. The mathematical model is described in Section 2. In Section 3, the local and parallel algorithm is given, and the convergence is deduced in Section 4. Finally, numerical experiments are presented.

2 Model Problem

We consider the model in a bounded domain $\Omega \subset R^d (d = 2 \text{ or } 3)$, which consists of a fluid flow region Ω_f and a porous media region Ω_p, see Fig. 1. Here $\Omega_f \cap \Omega_p = \emptyset$, $\overline{\Omega}_f \cup \overline{\Omega}_p = \overline{\Omega}$. Two domains are separated by the interface $\Gamma = \partial\Omega_f \cap \partial\Omega_p$. Denote $\Gamma_f = \partial\Omega_f \setminus \Gamma$, $\Gamma_p = \partial\Omega_p \setminus \Gamma$.

The Navier-Stokes equations for the fluid velocity $u(x)$ and pressure $p(x)$ describe the fluid flow in Ω_f

$$-\nu\Delta u + (u\cdot\nabla)u + \nabla p = f_1 \quad \text{in } \Omega_f, \tag{2.1}$$

$$\nabla\cdot u = 0 \quad \text{in } \Omega_f, \tag{2.2}$$

here ν is the kinetic viscosity, f_1 is the external force acting on the fluid flow.