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POSITIVE STEADY STATES OF A DIFFUSIVE PREDATOR-PREY SYSTEM WITH PREDATOR CANNIBALISM*

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Abstract The purpose of this paper is to investigate positive steady states of a diffusive predator-prey system with predator cannibalism under homogeneous Neumann boundary conditions. With the help of implicit function theorem and energy integral method, non-existence of non-constant positive steady states of the system is obtained, whereas coexistence of non-constant positive steady states is derived from topological degree theory. The results indicate that if dispersal rate of the predator or prey is sufficiently large, there is no non-constant positive steady states. However, under some appropriate hypotheses, if the dispersal rate of the predator is larger than some positive constant, for certain ranges of dispersal rates of the prey, there exists at least one non-constant positive steady state.

Key words Predator-prey; existence and nonexistence; pattern formation

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1 Introduction

A fundamental goal of theoretical ecology is to understand how the interactions of individual organism with each other and with environment determine the distribution of populations and the structure of communities. One way to understand how dispersal and spatial effects influence populations and communities is by using reaction-diffusion models, since reaction-diffusion models provide a way to translate local assumptions or data about the movement, mortality, and reproduction of individuals into global conclusions about the persistence or extinction of populations and the coexistence of interacting species.

In this paper, we consider a reaction-diffusion system modelling predator-prey interactions with the following form

$$\begin{cases} \dfrac{\partial u}{\partial t} = D_u\Delta u + ru\Big(1-\dfrac{u}{K}\Big) - \dfrac{auv}{1+ahu} & \text{in } \Omega\times(0,\infty), \\ \dfrac{\partial v}{\partial t} = D_v\Delta v + \dfrac{abuv}{1+ahu} - dv - \dfrac{av^2}{1+ahu} & \text{in } \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega\times(0,\infty), \\ u(x,0)=u_0(x), \quad v(x,0)=v_0(x) & \text{in } \Omega, \end{cases} \tag{1.1}$$

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where $u(x,t)$ and $v(x,t)$ account for the population density of the prey and predator, respectively, at time t and position x, and are therefore assumed to be nonnegative, with corresponding migration rates D_u and D_v. For simplicity we assume that both $u_0(x)$ and $v_0(x)$ are nonnegative and not identical to zero. The constant K denotes intrinsic growth rate or carrying capacity of the prey population, r is the growth rate, b is the food-to-newborn conversion factor, d is death rate of the predator, a is the scalings of the predator-prey encounter rate, h stands for the handling time, and $av^2/(1+ahu)$ represents predator cannibalism. $\Delta := \sum\limits_{i=1}^{N} \partial^2/\partial x_i^2$ denotes the Laplace operator in $\mathbb{R}^N$ which describes the random motion of the predator and prey, the habitat Ω is a bounded region in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$; The homogeneous Neumann boundary condition means that no individual crosses the boundary of the habitat; $\partial_\nu = \nu \cdot \nabla$, where ν denotes the outward unit normal vector on $\partial\Omega$. The reaction term is a Holling type II function response. We shall assume that D_u, D_v, r, K, a, b and d are all positive constants for the rest of the paper.

Through some kind of scaling,

$$
\begin{aligned}
&\overline{u} = ahu, \quad \overline{v} = \frac{av}{r}, \quad \overline{t} = tr, \quad l = \frac{b}{hr},\\
&m = ahK, \quad s = \frac{d}{r}, \quad D_1 = \frac{D_u}{r}, \quad D_2 = \frac{D_v}{r}
\end{aligned}
$$

for simplicity we drop the bar, system (1.1) is reduced to

$$
\begin{cases}
\dfrac{\partial u}{\partial t} = D_1 \Delta u + u\Big(1 - \dfrac{u}{m}\Big) - \dfrac{uv}{1+u} & \text{in } \Omega \times (0,\infty),\\
\dfrac{\partial v}{\partial t} = D_2 \Delta v + \dfrac{luv}{1+u} - sv - \dfrac{v^2}{1+u} & \text{in } \Omega \times (0,\infty),\\
\dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0,\infty),\\
u(x,0) = u_0(x), \quad v(x,0) = v_0(x) & \text{in } \Omega,
\end{cases} \tag{1.2}
$$

where m, l, s, D_1 and D_2 are all positive constants. For the biological point of view, we are mainly interested in the positive steady states of (1.2), since the asymptotical behaviors of positive solutions of (1.2) are closed to the positive steady states of (1.2), we shall investigate the following system

$$
\begin{cases}
-D_1 \Delta u = u\Big(1 - \dfrac{u}{m}\Big) - \dfrac{uv}{1+u} & \text{in } \Omega,\\
-D_2 \Delta v = \dfrac{luv}{1+u} - sv - \dfrac{v^2}{1+u} & \text{in } \Omega,\\
\dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega.
\end{cases} \tag{1.3}
$$

The positive steady state (u,v) of (1.1) refers to a classical one. Disregarding the diffusion in (1.3), the corresponding ODE system has three nonnegative equilibrium points: (i) $E_1(0,0)$, which corresponds to total extinction of the species; (ii) $E_2(m,0)$, which corresponds to extinction of the predator; (iii) $E^* = (u^*, v^*)$, which corresponds to coexistence of the predator and

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