Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2017,**37B**(5):1536–1544
Mathematica Acta Scientia 数学物理学报
http://actams.wipm.ac.cn

UNIVERSAL INEQUALITIES FOR A HORIZONTAL LAPLACIAN VERSION OF THE CLAMPED PLATE PROBLEM ON CARNOT GROUP*

Feng DU (杜锋)
School of Mathematics and Physics Science, Jingchu University of Technology, Jingmen 448000, China
E-mail: defengdu123@163.com

Chuanxi WU (吴传喜)
Faculty of Mathematics and Statistics, Hubei University, Wuhan 430062, China
E-mail: cxwu@hubu.edu.cn

Guanghan LI (李光汉)
School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China
E-mail: liguanghan@163.com

Changyu XIA (夏昌玉)
Departamento de Matemática, Universidade de Brasilia, 70910-900-Brasilia-DF, Brazil
E-mail: xia@mat.unb.br

Abstract In this paper, we investigate a horizontal Laplacian version of the clamped plate problem on Carnot groups and obtain some universal inequalities. Furthermore, for the lower order eigenvalues of this eigenvalue problem on carnot groups, we also give some universal inequalities.

Key words eigenvalue; universal inequality; horizontal Laplacian; Carnot group

2010 MR Subject Classification 35P15; 53C20; 53C42

1 Introduction and Main Results

Let Ω be a bounded domain in an n-dimensional complete Riemannian manifold M. Let Δ be the Laplace operator acting on functions on M and consider the following eigenvalue problem for the biharmonic operator

$$\begin{cases} \Delta^2 u = \lambda u, & \text{in } \Omega, \\ u = \dfrac{\partial u}{\partial \nu} = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

*Received January 8, 2014, 2014; revised May 2, 2017. This research was partially supported by the NSF of China (11171096, 11401131), NSF of Hubei Provincial Department of Education (Q20154301), and CNPq, Brazil.

where ν denotes the outward unit normal vector field of $\partial\Omega$. Problem (1.1) is also called clamped plate problem which describes the characteristic vibrations of a clamped plate. It is known that this eigenvalue problem has a discrete spectrum,

$$0 < \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_k \leq \cdots ,$$

where each eigenvalue is repeated with its multiplicity. For the eigenvalues of the clamped plate problem (1.1), some universal inequalities were given by some mathematicians (see [2, 3, 9, 11, 13, 14]).

In recent years, some eigenvalue problems on the Heisenberg group were studied by many people (see [5, 6, 8, 10, 12]), they got some universal bounds for eigenvalues of these problems. Since the Heisenberg group $\mathbb{H}^n$ is the simplest example of a Carnot group of step 2, it is natural to consider the following problem.

Problem 1 For the eigenvalue problem of a sub-Laplacian on a Carnot group, does there exist a universal bounds on the $(k+1)$-th eigenvalue in terms of the first k eigenvalues?

With respect to the above problem, Aribi and El Soufi [1] gave the universal bounds for the eigenvalues of the horizontal Laplacian on Carnot groups. Inspired by these investigations, we will consider the horizontal Laplacian version clamped plate problem on Carnot groups. Following, we give a short introduction for Carnot groups.

A r-step Carnot group is a connected, simply connected, nilpotent Lie group G whose Lie algebra $\mathfrak{g}$ admits a stratification $\mathfrak{g}= H_1 \oplus \cdots \oplus H_r$, so that $[H_1, H_j] = H_{j+1}, j = 1, \cdots , r-1$ and $[H_i, H_j] \subset H_{i+j}$, $j = 1, \cdots , r$ with $H_k = 0$ for $k > r$, where $[\cdot, \cdot]$ denotes the Lie brackets and each H_i is a vector subspace of $\mathfrak{g}$. We also assume that $\mathfrak{g}$ carries a scalar product $\langle , \rangle_g$ for which the subspaces H_j are mutually orthogonal. The layer H_1 generates the whole $\mathfrak{g}$ and induces a sub-bundle HG of TG of rank $d_1 = \dim_{H_1}$ that we call the horizontal bundle of the Carnot group. Following, we introduce a sub-Laplacian on a Carnot group G. For each $i \leq r$, let $\dim H_i = d_i$, and $\{e^i_1, \cdots , e^i_{d_i}\}$ be an orthonormal basis of H_i and denote by $\{X^i_1, \cdots , X^i_{d_i}\}$ the system of left invariant vector fields that coincides with $\{e^i_1, \cdots , e^i_{d_i}\}$ at the identity element of G. We consider the Riemannian metric g_G on G with respect to which the family $\{X^1_1, \cdots , X^1_{d_1}, \cdots , X^r_1, \cdots , X^r_{dr}\}$ constitute an orthonormal frame for TG. The corresponding Levi-Civita connection ∇ induces a connection ∇^H on HG that we call horizontal connection: if X and Y are smooth sections of HG, then $\nabla^H_X Y = \pi_H \nabla_X Y$, where $\pi_H : TG \to HG$ is the orthogonal projection. The horizontal Laplacian Δ_H is then defined for every C^2 function f on G by

$$\Delta_H f := \text{trace}_H \nabla_H du = \sum_{i=1}^{d_1} (X^1_i)^2(f).$$

The operator Δ_H is a hypoelliptic operator of Hörmander type.

Let Ω be a bounded domain in a Carnot group G, and let Δ_H be the horizontal Laplace operator acting on functions on G and consider the following eigenvalue problem

$$\begin{cases} \Delta^2_H u = \lambda u, & \text{in } \Omega, \\ u = \dfrac{\partial u}{\partial \nu} = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

where ν is the outward unit normal vector field of the boundary $\partial\Omega$ of Ω.