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DYNAMICAL BEHAVIOR OF A STOCHASTIC HBV INFECTION MODEL WITH LOGISTIC HEPATOCYTE GROWTH*

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Abstract This paper is concerned with a stochastic HBV infection model with logistic growth. First, by constructing suitable stochastic Lyapunov functions, we establish sufficient conditions for the existence of ergodic stationary distribution of the solution to the HBV

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infection model. Then we obtain sufficient conditions for extinction of the disease. The stationary distribution shows that the disease can become persistent in vivo.

Key words stochastic HBV infection model; extinction; stationary distribution; Lyapunov function

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1 Introduction

Nowadays, hepatitis B virus (HBV) is one of the major diseases in the world. The World Health Organization (WHO) reports that over one-third of the world's population has been or is actively infected by HBV, more than 400 million are chronic infectious, and 25-40 percent of these chronic infection carriers will die from liver cirrhosis or primary hepatocellular carcinoma. The HBV carrier rate varies from 0.1 percent to 20 percent in different areas of the world [1].

In order to describe the virus dynamics in vivo and by considering the ability of regeneration of the hepatocyte possess [2], Li et al. [3] established a HBV infection model with logistic hepatocyte growth which takes the following form

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = r_1 x\Big(1 - \dfrac{x+y}{K}\Big) - \beta x v, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \beta x v - a_1 y, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = k_1 y - \gamma v, \end{cases} \tag{1.1}$$

where $x = x(t)$, $y = y(t)$ and $v = v(t)$ are the numbers (densities) of healthy hepatocyte, infected hepatocyte, and viral particles (virions) at time t, respectively. Healthy hepatocyte are assumed to become infected at the rate of βxv, where β is a constant rate describing the infection process. Infected hepatocyte are lost at a rate of $a_1 y$, and virus are produced by infected hepatocyte at a rate of $k_1 y$ and removed at a rate of γv, r_1 is the maximal regeneration rate per healthy hepatocyte, and K is the carrying capacity.

By making the change of variables

$$x = K\bar{x},\ y = K\bar{y},\ v = \frac{\gamma\bar{v}}{\beta},\ t = \frac{\bar{t}}{\gamma},$$

and then dropping the bars in system (1.1), Li et al. [3] studied the following dimensionless system

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = rx(1 - x - y) - xv, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = xv - ay, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = ky - v, \end{cases} \tag{1.2}$$

where

$$r = \frac{r_1}{\gamma},\ a = \frac{a_1}{\gamma},\ k = \frac{\beta k_1 K}{\gamma^2}.$$

In system (1.2), the basic reproduction number is $R_0 = \frac{k}{a}$ which determines the extinction and persistence of the disease.