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ON THE GLOBAL EXISTENCE OF SMOOTH SOLUTIONS TO THE MULTI-DIMENSIONAL COMPRESSIBLE EULER EQUATIONS WITH TIME-DEPENDING DAMPING IN HALF SPACE*

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Abstract This paper is a continue work of [4, 5]. In the previous two papers, we studied the Cauchy problem of the multi-dimensional compressible Euler equations with time-depending damping term $-\frac{\mu}{(1+t)^\lambda}\rho u$, where $\lambda \geq 0$ and $\mu > 0$ are constants. We have showed that, for all $\lambda \geq 0$ and $\mu > 0$, the smooth solution to the Cauchy problem exists globally or blows up in finite time. In the present paper, instead of the Cauchy problem we consider the initial-boundary value problem in the half space $\mathbb{R}^d_+$ with space dimension $d = 2, 3$. With the help of the special structure of the equations and the fluid vorticity, we overcome the difficulty arisen from the boundary effect. We prove that there exists a global smooth solution for $0 \leq \lambda < 1$ when the initial data is close to its equilibrium state. In addition, exponential decay of the fluid vorticity will also be established.

Key words compressible Euler equations; initial-boundary value problem; damping; global existence

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1 Introduction

In this paper, we are concerned with the global existence of smooth solution to the initial-boundary value problem of the multi-dimensional compressible Euler equations with time-depending damping in the half space $\mathbb{R}^d_+$

$$\begin{cases} \partial_t\rho + \operatorname{div}(\rho u) = 0, \\ \partial_t(\rho u) + \operatorname{div}(\rho u \otimes u + p(\rho)\mathrm{I}_d) = -\dfrac{\mu}{(1+t)^\lambda}\rho u, \\ \rho(0,x) = \bar\rho + \rho_0(x), \quad u(0,x) = u_0(x), \\ u\cdot\vec{n}|_{\partial\mathbb{R}^d_+} = -u_d(t,x',0) = 0, \quad (t,x') \in \mathbb{R}_+ \times \mathbb{R}^{d-1}, \end{cases} \tag{1.1}$$

where $x = (x_1, \cdots, x_d) \overset{\text{def}}{=} (x', x_d) \in \mathbb{R}^d_+ \overset{\text{def}}{=} \mathbb{R}^{d-1} \times \mathbb{R}_+$ with space dimension $d = 2, 3$ and $\vec{n} = (0, \cdots, -1)$ is the unit outward normal of $\mathbb{R}^d_+$. ρ, $u = (u_1, \cdots, u_d)$ and $p(\rho)$ stand for the

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density, velocity and pressure, respectively, I_d is the $d \times d$ identity matrix. The state equation of the gases is assumed to be $p(\rho) = A\rho^\gamma$, where $A > 0$ and $\gamma > 1$ are constants. Furthermore, $\mu > 0$, $\lambda \geq 0$, $\bar{\rho} > 0$ are constants, ρ_0, $u_0 \in C_0^\infty(\overline{\mathbb{R}_+^d})$, $(\rho_0, u_0) \not\equiv 0$, and $\rho(0, x) > 0$.

If $\lambda = 0$ and $\mu > 0$, the system (1.1) admits global solutions. In one dimension, there are many results on this subject. In the isentropic case, the readers are referred to [6, 15] for the asymptotic behavior of solutions to the Cauchy problem and the initial-boundary value problem; to [10] for the Riemann problem; to [16] for the global L^∞ entropy weak solutions to the initial-boundary value problem; to [2] for the BV solutions. For the non-isentropic flows, the Cauchy problem and the initial-boundary value problem have been studied in [8, 9].

In multi-dimensions, the global existence of the Cauchy problem and the long-term behavior of the solution were established in [20–23] and the references therein. Also in multi-dimensions, Fang and Xu [3] considered the C^1 solutions to the Cauchy problem on the framework of Besov space. In [13], Lu studied the exponential stability of constant steady state on torus in arbitrary dimensions. As far as we know, there are few works on the initial-boundary value problem in multi-dimensions, see [12, 17, 24]. The main difficulty lies in the boundary condition, while in one dimension this condition can be viewed as the Dirichlet condition.

For $\mu > 0$ and $\lambda > 0$, the authors in [4, 5] studied the Cauchy problem in two or three space dimensions

$$\begin{cases} \partial_t\rho + \mathrm{div}(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u + p(\rho)\mathrm{I}_d) = -\dfrac{\mu}{(1+t)^\lambda}\rho u, \\ \rho(0, x) = \bar{\rho} + \epsilon\tilde{\rho}_0(x), \quad u(0, x) = \epsilon\tilde{u}_0(x), \end{cases} \tag{1.2}$$

where $x = (x_1, \cdots, x_d) \in \mathbb{R}^d$ with $d = 2, 3$ and $\epsilon > 0$ is a small constant. The authors established the following systematic results

$$\begin{cases} T_\epsilon = \infty, & \text{if } 0 \leq \lambda < 1, \text{ or } \lambda = 1,\ \mu > 3 - d \text{ with } \mathrm{curl}\, \tilde{u}_0 \equiv 0, \\ T_\epsilon < \infty, & \text{if } \lambda > 1 \text{ or } \lambda = 1,\ 0 < \mu \leq 3 - d, \end{cases}$$

where T_ϵ is the lifespan of small amplitude smooth solution (ρ, u) to the Cauchy problem (1.2).

In the present paper, we will study the initial-boundary value problem to (1.1) for $\lambda \geq 0$ in the half space $\mathbb{R}_+^d$. Due to the degeneracy of the time-depending damping $-\frac{\mu}{(1+t)^\lambda}\rho u$, the approach in [12, 17, 24] cannot be applied directly to our problem, see Remark 4.2.

Through out this paper, $\|\Phi(t, \cdot)\| \overset{\text{def}}{=} \|\Phi(t, x)\|_{L_x^2(\mathbb{R}_+^d)}$ and $\|\Phi(t, \cdot)\|_j \overset{\text{def}}{=} \|\Phi(t, \cdot)\|_{H^j} = \sum\limits_{i \leq j} \|\nabla_x^i \Phi(t, \cdot)\|$ denote the norms of $L^2(\mathbb{R}_+^d)$ and the Sobolev space $H^j(\mathbb{R}_+^d)$, respectively. In addition, we define

$$X_4([0, T] \times \mathbb{R}_+^d) \overset{\text{def}}{=} \{\Phi : \partial_t^l \Phi \in L^\infty([0, T]; H^{4-l}(\mathbb{R}_+^d)), l = 0, 1, \cdots, 4\}.$$

Now, we state the main result in this paper.

Theorem 1.1 Suppose that the initial data satisfies the compatibility condition $\partial_t^l u_d(0, x', 0) = 0$, $l = 1, \cdots, 4$. For all $\mu > 0$ and $0 \leq \lambda < 1$, there exists a small constant $\varepsilon_0 > 0$ such that if the initial data satisfies $\|(\rho - \bar{\rho}, u)(0, \cdot)\|_4 = \varepsilon \leq \varepsilon_0$, then (1.1) admits a global solution $(\rho, u) \in C^1([0, +\infty) \times \mathbb{R}_+^d) \cap X_4([0, +\infty) \times \mathbb{R}_+^d)$ which satisfies $\rho(t, x) > 0$ and

$$\|(\rho - \bar{\rho}, u)(t, \cdot)\|^2 + \sum_{1 \leq i+j \leq 4} (1+t)^{2\lambda} \|\partial_t^i \nabla_x^j (\rho, u)(t, \cdot)\|^2 + \int_0^t (1+s)^{-\lambda} \|u(s, \cdot)\|^2 \mathrm{d}s$$

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