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BLOW-UP AND LIFE SPAN ESTIMATES FOR A CLASS OF NONLINEAR DEGENERATE PARABOLIC SYSTEM WITH TIME-DEPENDENT COEFFICIENTS*

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Abstract This paper deals with the singularity and global regularity for a class of nonlinear porous medium system with time-dependent coefficients under homogeneous Dirichlet boundary conditions. First, by comparison principle, some global regularity results are established. Secondly, using some differential inequality technique, we investigate the blow-up solution to the initial-boundary value problem. Furthermore, upper and lower bounds for the maximum blow-up time under some appropriate hypotheses are derived as long as blow-up occurs.

Key words porous medium systems; Dirichlet boundary conditions; global existence; blow-up; upper and lower bounds

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1 Introduction and Main Results

In this article, we deal with the following porous medium system with time-dependent coefficients under homogeneous Dirichlet boundary conditions

$$\begin{cases} u_t = \Delta u^m + f_1(t)g_1(v), & (x,t) \in \Omega \times (0,T), \\ v_t = \Delta v^n + f_2(t)g_2(u), & (x,t) \in \Omega \times (0,T), \\ u(x,t) = v(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

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where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, and $m, n > 1$. The coefficients $f_1(t)$, $f_2(t)$ are positive continuous functions for any $t > 0$. The nonlinearities $g_1(v)$, $g_2(u)$ are assumed to satisfy $g_1(v) > 0 (v > 0)$, $g_2(u) > 0 (u > 0)$ and $g_1(0) = g_2(0) = 0$. The initial value $u_0(x)$, $v_0(x)$ are nontrivial nonnegative continuous functions and vanish on $\partial\Omega$. More specific assumptions on the data will be made later.

Global existence and blow-up to the nonlinear parabolic equation have been investigated extensively, please see the surveys [21], [14], [20] and [22]. In this paper, we want to derive conditions on the data of problem (1.1) sufficient to ensure the global existence of the solution, investigate the solution which blows up in finite time under some assumptions and estimate the life span of the singular solution.

We mention in particular the early works by Fujita [11], Weissler [23] and Weigner [24]. In recent papers, Payne and Philippin [18] derived blow-up time estimates in a system where the equations have linear diffusion terms under Dirichlet boundary conditions, while in [15] Marras and Vernier-Piro considered a system with linear diffusion terms and nonlocal boundary conditions. We would like to draw lessons from some results on blow-up solutions to the degenerate parabolic equations or system in [2–10], [13], and [16, 17].

Recently, in [25], Xia-Pu-Li obtained some results on the global existence and blow-up of the classical solutions to a special case of (1.1).

First, we obtain some results on the global existence of the classical solution to problem (1.1).

Theorem 1.1 Assume that there exist positive constants $p, q, \bar{k}_1$ and $\bar{k}_2$, such that $g_1(\xi) \le \xi^p, g_2(\xi) \le \xi^q$ for $\xi > 0$ and $f_1(t) \le \bar{k}_1$, $f_2(t) \le \bar{k}_2$ for any $t > 0$. If $pq < mn$, then every classical solution to the initial-boundary value problem (1.1) is global.

Second, we get the blow-up results to problem (1.1) in the following Theorem 1.2 and Theorem 1.3. We also obtain the upper bound for the blow-up time T in Theorem 1.3.

Theorem 1.2 Assume that there exist positive constants p, q, such that $g_1(\xi) \ge \xi^p$, $g_2(\xi) \ge \xi^q$ for $\xi > 0$, and $\underline{k} := \min\{\inf f_1(t), \inf f_2(t)\} > 0$. If $pq > mn$, then the classical solution to the initial-boundary value problem (1.1) blows up in finite time T for large data $u_0(x), v_0(x)$.

Theorem 1.3 Assume that there exist positive constants p, q, such that $g_1(\xi) \ge \xi^p$, $g_2(\xi) \ge \xi^q$ for $\xi > 0$, and $\underline{k} := \min\{\inf f_1(t), \inf f_2(t)\}$. If $q \ge m, p \ge n$, $\underline{k} > \lambda_1$, then the classical solution to the initial-boundary value problem (1.1) blows up in finite time T for large data $u_0(x), v_0(x)$, where λ_1 is the first eigenvalue to following problem

$$\begin{cases} \Delta\phi + \lambda\phi = 0, & x \in \Omega, \\ \phi = 0, & x \in \partial\Omega \end{cases} \tag{1.2}$$

with $\phi > 0$ and $\int_\Omega \phi \mathrm{d}x = 1$. Moreover, there exists a $\bar{T}_0 > 0$, which depends on m, n, p, q, $\underline{k}$ and the initial data $u_0(x), v_0(x)$, such that $T \le \bar{T}_0$.

Finally, we get the lower bound for the blow-up time T as long as blow-up occurs.

Theorem 1.4 Suppose that Ω be a domain in $\mathbb{R}^3$ and there exist two positive constants p, q, such that $g_1(\xi) \le \xi^p, g_2(\xi) \le \xi^q$ for $\xi > 0$. If the solution (u, v) of (1.1) blows up in finite time T, then there exists a positive constant $\underline{T}_0$ such that $T \ge \underline{T}_0$, where $\underline{T}_0$ depends on