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IMPULSIVE DIFFERENTIAL EQUATIONS WITH GAMMA DISTRIBUTED MOMENTS OF IMPULSES AND P-MOMENT EXPONENTIAL STABILITY*

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Abstract Differential equations with impulses at random moments are set up and investigated. We study the case of Gamma distributed random moments of impulses. Several properties of solutions are studied based on properties of Gammma distributions. Some sufficient conditions for p-moment exponential stability of the solutions are given.

Key words impulses; Gamma distributed moments of impulses; p-moment exponential stability; Lyapunov functions

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1 Introduction

Modeling real world phenomena where the state of the process under investigation changes instantaneously at uncertain moments leads to combining ideas in differential equations and probability theory. When there is uncertainty in the behavior of the state of the investigated process an appropriate model is usually a stochastic differential equation where one or more of the terms in the differential equation are stochastic processes, and this usually results with the solution being a stochastic process ([10–13]). Many real world phenomena are characterized by deterministic changes of the state of the process under investigations between two consecutive instantaneous changes at uncertain moments. In this case appropriate models are impulsive

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differential equations with random impulses. The presence of random variables usually determine that the solutions of these equations are stochastic processes. We note that impulsive differential equations with random impulsive moments differs from stochastic differential equations with jumps. One of the main practical as well as theoretical questions is stability, i.e., can one preserve the closeness of two solutions if they start off close? Deterministic differential equations with random impulses were considered, for example, in [3, 4, 8] and a fractional differential equation with random Erlang distributed moments of impulses was studied in [9]. In these papers there are some inaccuracies in properties of random variables, stochastic processes and deterministic variables, for example, in the convergence of a sequence of random variables, the convergence of a function when its real valued argument approaches a random variable ([9, p.430]), and the restriction $d_k < \infty$ when the values of the Erlang distribution are in $(0, \infty)$.

The main goal of the paper is to study stability properties of solutions of impulsive differential equation when the waiting time between two consecutive impulses is Gamma distributed. The p-moment exponential stability of the solutions is defined and studied. Initially some properties of the solution with Gamma distributed impulses are studied and then some sufficient conditions for p-moment exponential stability are given.

2 Random Impulses in Differential Equations

Initially, we will give a brief overview of differential equations with deterministic impulses (for more details see [5–7] and the cited references therein. A survey on deterministic impulses in fractional differential equations was given in [2]).

Let the increasing sequence of nonnegative points $\{T_k\}_{k=0}^{\infty}$ be given with $\lim\limits_{k\to\infty}\{T_k\} = d \leq \infty$. Consider the initial value problem for the system of impulsive differential equations (IDE) with fixed points of impulses

$$\begin{aligned} &x' = f(t, x(t)) \quad \text{for} \quad t \in (T_k, T_{k+1}], \quad k = 0, 1, 2, \cdots, \\ &x(T_k + 0) = I_k(x(T_k - 0)) \quad \text{for} \ \ k = 1, 2, \cdots, \\ &x(T_0) = x_0, \end{aligned} \tag{2.1}$$

where $x, x_0 \in \mathbb{R}^n$, $f : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$, $I_k : \mathbb{R}^n \to \mathbb{R}^n$.

The solution of IDE (2.1) depends not only on the initial condition (T_0, x_0) but on the moments of impulses T_k, $k = 1, 2, \cdots$ and we denote it by $x(t; T_0, x_0, \{T_k\})$. We assume that $x(T_k; T_0, x_0, \{T_k\}) = \lim\limits_{t\to T_k-0} x(t; T_0, x_0, \{T_k\})$ for any $k = 1, 2, \cdots$.

We assume the following conditions are satisfied.

H1 $f(t, 0) = 0$ and $I_k(0) = 0$ for $t \geq 0$, $k = 1, 2, \cdots$.

H2 For any initial value (T_0, x_0) the ODE $x' = f(t, x)$ with $x(T_0) = x_0$ has a unique solution $x(t) = x(t; T_0, x_0)$ defined for $t \in [T_0, d)$.

Let the probability space $(\Omega, \mathcal{F}, P)$ be given, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of Ω, and P is the probability function. Let $\{\tau_k\}_{k=1}^{\infty}$ be a sequence of random variables defined on the probability space $(\Omega, \mathcal{F}, P)$.

Assume $\sum\limits_{k=1}^{\infty} \tau_k = \infty$ with probability 1.

Remark 2.1 The random variables τ_k define the time between two consecutive impulsive moments of the impulsive differential equation with random impulses.