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ANALYSIS OF AN ELASTO-PIEZOELECTRIC SYSTEM OF HEMIVARIATIONAL INEQUALITIES WITH THERMAL EFFECTS*

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Abstract In this paper we prove the existence and uniqueness of a weak solution for a dynamic electo-viscoelastic problem that describes a contact between a body and a foundation. We assume the body is made from thermoviscoelastic material and consider nonmonotone boundary conditions for the contact. We use recent results from the theory of hemivariational inequalities and the fixed point theory.

Key words dynamic contact; evolution hemivariational inequality; Clarke subdifferential; nonconvex; parabolic; viscoelastic material; frictional contact; weak solution; piezoelectricity

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1 Introduction

Problems involving piezoelectricity arise when there is a coupling between the mechanical and electrical properties of the materials. The mechanical stress generates the electric potential and also the mechanical stress is influenced, when the electric potential is applied. Piezoelectric materials are use in radioelectronics, electroacustics. On the other hand, thermoviscoelastic material, where the mechanical stress is influenced by the change of the temperature of the material appear in multitude of industrial problems.

The contact problems with piezoelectric contact have been studied in many papers, see [12, 14–16, 26] and references therein. For general books on piezoelectric effect we refer to [9, 27]. On the other hand, thermoviscoelastic problems have been studied extensively in [4, 7, 8, 21]. The models describing both piezoelectric and thermal effects are new from the mathematical point of view, the recent papers investigating this kind of bodies are [2, 3]. We generalize these results by considering nonmononote boundary conditions described by the Clarke subdifferential and using the theory of hemivariational inequalities. We also consider a parabolic equation for the temperature, instead of elliptic. It is worth noting that there are many papers concerning

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this thermo-electo-viscoelastic phenomena from physical and numerical point of view, see [1, 10] and references therein.

We note that the notion of hemivariational inequality was introduced by Panagiotopoulos in [24, 25]. Since then hemivariational inequality proved to be an efficient tool to solve important and open problems in mechanics. The mathematical theory of stationary hemivariational inequalities can be found in Naniewicz and Panagiotopoulos [23] and the references therein. Dynamic hemivariational inequalities modelling viscoelastic contact problems for various materials have been studied, for instance, in [13, 19, 20, 28]. Recent results on nonlinear stationary and evolutionary inclusions and hemivariational inequalities, and their applications to contact mechanics, can be found in [17].

The present paper is a contiunation of [21] and deals with a new mathemtical model describing the contact between an electro-viscoelestic body with thermal effects and the conductive foundation. The contact is described by possibly nonmonotone subdifferential conditions describing the friction, heat flux and electicic conductivity. We underline that in the model there is a strong coupling not only in the constitutive relations but also in the heat exchange boundary condition which depends on the tangential component of the velocity and in the electric displacement boundary condition which depends on the normal velocity. We show that the resulting model is well-posed from mathematical point of view.

The paper is organized as follows. In Section 2 we introduce some notation and preliminaries. In Section 3, we present the model of frictional contact between the electro-thermo-viscoelastic body and a conductive foundation. In Section 4 we introduce the variational formulation of the problem, given as a system of three hemivariational inequalities. We provide the assumptions on the data and state the main theorem of the paper on existence and uniqueness of a weak solution to the problem, together with the proof. The proof in Section 4 is based on an abstract theorem on evolutionary inclusion and fixed point theorems.

2 Preliminaries

In this section we introduce notation and recall some definitions and results needed in the sequel. For details, see [6, 11, 29]. Let $d = 2, 3$. We denote by $\mathbb{S}^d$ the space $\mathbb{R}_s^{d\times d}$ of symmetric matrices of order d endowed with canonical inner product defined by $\sigma\colon \tau = \sigma_{ij}\tau_{ij}$ for $\sigma, \tau \in \mathbb{S}^d$. We use the summation convention over repeated indices from 1 to d.

Let Ω be an open bounded subset of $\mathbb{R}^d$ with a Lipschitz continuous boundary Γ. We introduce the spaces

$$H = L^2(\Omega;\mathbb{R}^d) \text{ and } \mathcal{H} = \left\{\tau = (\tau_{ij}) \mid \tau_{ij} = \tau_{ji} \in L^2(\Omega)\right\}.$$

By $\varepsilon\colon H^1(\Omega;\mathbb{R}^d) \to \mathcal{H}$ we denote the deformation operator given by

$$\varepsilon(u) = \big(\varepsilon_{ij}(u)\big), \quad \varepsilon_{ij}(u) = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big).$$

For $v \in L^2(\Gamma;\mathbb{R}^d)$ we denote by v_ν and v_τ the usual normal and tangential components of v on the boundary Γ, i.e., $v_\nu = v\cdot\nu$ and $v_\tau = v - v_\nu\nu$, where ν denotes the normal outward unit vector. Similarily, for a regular tensor field $\sigma\colon \Omega \to \mathbb{S}^d$, we define its normal and tangential components by $\sigma_\nu = (\sigma\nu)\cdot\nu$ and $\sigma_\tau = \sigma\nu - \sigma_\nu\nu$, respectively.