Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2017,**37B**(4):1133–1150
Acta Mathematica Scientia 数学物理学报
http://actams.wipm.ac.cn

OPTIMALITY CONDITIONS AND DUALITY RESULTS FOR NONSMOOTH VECTOR OPTIMIZATION PROBLEMS WITH THE MULTIPLE INTERVAL-VALUED OBJECTIVE FUNCTION*

Tadeusz ANTCZAK
Faculty of Mathematics and Computer Science, University of Łódź, Banacha 22, 90-238 Łódź, Poland
E-mail: *antczak@math.uni.lodz.pl*

Abstract In this paper, both Fritz John and Karush-Kuhn-Tucker necessary optimality conditions are established for a (weakly) *LU*-efficient solution in the considered nonsmooth multiobjective programming problem with the multiple interval-objective function. Further, the sufficient optimality conditions for a (weakly) *LU*-efficient solution and several duality results in Mond-Weir sense are proved under assumptions that the functions constituting the considered nondifferentiable multiobjective programming problem with the multiple interval-objective function are convex.

Key words nonsmooth multiobjective programming problem with the multiple interval-objective function; Fritz John necessary optimality conditions; Karush-Kuhn-Tucker necessary optimality conditions; (weakly) *LU*-efficient solution; Mond-Weir duality

2010 MR Subject Classification 90C25; 90C29; 90C30; 90C46

1 Introduction

Nonlinear multiobjective programming problems, also known as vector optimization problems, occur in various fields of applications in Operational Research. Multiobjective programming problems are those where two or more objectives are to be minimized on some set of all feasible solutions. In such problems, we deal with conflicts amongst objectives.

One of deterministic optimization models to deal with the extremum problems with uncertain data is interval-valued optimization because it does not require the specification or the assumption of probabilistic distributions (as in stochastic programming) or possibilistic distributions (as in fuzzy programming). The interval-valued optimization problems are closely related to inexact linear programming problems. Charnes et al. [7] considered the linear programming problems in which the right-hand sides of linear inequality constraints were taken as closed intervals. In [4], Bitran proved the connectedness of the set of efficient extreme points and the existence of efficient points in the multiobjective linear programming problems with

*Received December 22, 2015; revised April 13, 2016.

interval coefficients, where an implicit enumeration algorithm was also proposed to obtain the solutions.

In recent years, therefore, attempts were made by several authors to prove optimality conditions and duality results for new classes of interval-valued optimization problems, including a larger and larger class of interval-valued multiobjective programming ones. Ishibuchi and Tanaka [10] considered multiobjective programming problems with interval-valued objective functions and proposed the ordering relation between two closed intervals by considering the maximization and minimization problems separately. Chanas and Kuchta [6] generalized the concept of optimality introduced by lshihuchi and Tanaka [10] for vector optimization problems with interval-valued objective functions to the case of the linear multiobjective programming problem with interval coefficients in the objective function based on preference relations between intervals. Urli and Nadeau [18] used an interactive method to solve the linear multiobjective programming problems with interval coefficients and they also proposed a methodology to transform a nondeterministic problem into a deterministic problem. Further, the Karush-Kuhn-Tucker necessary optimality conditions for the differentiable optimization problems with the interval-valued single-objective function were investigated by Wu [19]. The Wolfe duality and Lagrangian duality for nonlinear interval-valued scalar optimization problems were discussed by Wu [20] and [23], respectively. In [22], Wu invoked the solution concept (Pareto optimal solution) in the (conventional) multiobjective programming problems to deal with the multiobjective programming problems with interval-valued objective functions. Under these settings, he derived the sufficiency of the Karush–Kuhn–Tucker optimality conditions in the considered multiobjective programming problem with interval-valued objective functions under convexity assumptions. Hosseinzade and Hassanpour [9] derived the Karush-Kuhn-Tucker optimality conditions in differentiable multiobjective programming problems with interval-valued objective and constraint functions under suitable convexity assumptions. They also obtained Pareto optimal solutions for such vector optimization problems by resorting to the sufficient optimality condition. In [24], Zhou and Wang derived optimality conditions and mixed duality results for considered scalar optimization problems with an interval-valued objective function. In [3], Bhurjee and Panda developed a methodology to study the existence of solutions in an interval optimization problem. Jana and Panda [11] studied the preferable efficient solutions of the considered interval-valued vector optimization problem. Jayswal et al. [12] established sufficient optimality conditions and appropriate duality theorems for Mond–Weir and Wolfe type duals for considered differentiable interval-valued programming problem under convexity hypotheses. In [15], for the considered interval-valued multiobjective problem, Preda established some necessary and sufficient optimality conditions for weak efficient solutions under new generalized convexities with the tool right-upper Dini derivative, which is an extension of directional derivative. Karmakar and Bhunia [13] proposed an alternative optimization technique via multiobjective programming for constrained optimization problems with interval-valued objectives. For the considered optimization problem with the interval-valued objective function, Chalco-Cano et al. [5] obtained the Karush-Kuhn-Tucker optimality conditions using the concept of generalized Hukuhara derivative (gH-derivative) for interval-valued functions.

In the recent years, many results to interval-valued optimization problems have been explored in the case when they are smooth, while only few papers studied the optimality conditions