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LIFE-SPAN OF CLASSICAL SOLUTIONS TO HYPERBOLIC GEOMETRY FLOW EQUATION IN SEVERAL SPACE DIMENSIONS*

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Abstract In this article, we investigate the lower bound of life-span of classical solutions of the hyperbolic geometry flow equations in several space dimensions with "small" initial data. We first present some estimates on solutions of linear wave equations in several space variables. Then, we derive a lower bound of the life-span of the classical solutions to the equations with "small" initial data.

Key words Hyperbolic geometry flow; classical solution; life-span

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1 Introduction

In this article, we shall consider the following hyperbolic geometry equation in several space dimensions

$$u_{tt} - \Delta \ln u = 0. \tag{1.1}$$

Equation (1.1) is first introduced by Kong, Liu, and Xu to describe the hyperbolic geometry flow on Riemannian surface. One can see [11] for the details.

Denote

$$\phi = \ln u, \tag{1.2}$$

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then equation (1.1) becomes

$$\phi_{tt} - e^{-\phi}\Delta\phi = -\phi_t^2. \tag{1.3}$$

Equation (1.3) is a quasilinear wave equation. In [11], Kong, Liu, and Xu investigate the solution of equation (1.3) in one space dimension. They prove that the solution can exist for all time by choosing a suitable initial velocity. On the other hand, if the initial velocity tensor does not satisfy the condition presented in [11], then the solution blows up at a finite time. Later, Kong, Liu, and Wang in [10] consider equation (1.3) in two space variables with decay initial data, and they give a lower bound of the life-span of classical solutions to the quasilinear wave equation.

In this article, we shall consider the Cauchy problem for (1.1) with $n(\geq 2)$ space variables with small initial data. In other words, we are interested in the Cauchy problem for equation (1.3) with the following initial data

$$t = 0 : \phi = \varepsilon\phi_0(x),\ \ \phi_t = \varepsilon\phi_1(x), \tag{1.4}$$

where $\phi_0(x), \phi_1(x) \in C^\infty(\mathbb{R}^n)$, and ε is a suitable small positive parameter.

The main results of this article can be described as follows.

Theorem 1.1 Suppose that $\phi_0(x), \phi_1(x) \in C_0^\infty(\mathbb{R}^n)$, then there exist two positive constants δ and ε_0 such that for any fixed $\varepsilon \in [0, \varepsilon_0]$, the Cauchy problem (1.3)–(1.4) has a unique solution on the interval $[0, T_\varepsilon]$, where T_ε is given by

$$T_\varepsilon \geq \begin{cases} \dfrac{\delta}{\varepsilon^2}, & n = 2, \\ e^{\frac{\delta}{\varepsilon}}, & n = 3, \\ \infty, & n \geq 4. \end{cases} \tag{1.5}$$

Theorem 1.2 Assume that $\phi_0(x), \phi_1(x) \in C^\infty(\mathbb{R}^3)$, and there exist two constants $K > 0$ and $k > \frac{3}{2}$ such that

$$|\phi_0(x)| \leq \frac{K}{(1+|x|)^{k+1}},\ \ |\phi_1(x)| \leq \frac{K}{(1+|x|)^{k+1}}, \tag{1.6}$$

then there exist two positive constants δ and ε_0 such that for any fixed $\varepsilon \in [0, \varepsilon_0]$, the Cauchy problem (1.3)–(1.4) has a unique C^∞ solution on the interval $[0, T_\varepsilon]$, where T_ε is given by

$$T_\varepsilon \geq \frac{\delta}{\varepsilon^2}. \tag{1.7}$$

Remark 1.3 Theorem 1.1 improve the result in [10].

This article is organized as follows. In Section 2, we establish some estimates on the solutions of linear wave equations in multiple space variables, which play an important role in the proof of Theorem 1.1 and Theorem 1.2. In Section 3, we devoted to the proof of Theorem 1.1 and Theorem 1.2 based on the estimates in Section 2.

2 Some Useful Lemmas

In this section, we give some Lemmas, which play an important role in the proof of Theorem 1.1. At first, we introduce a set of Klainerman's invariant vector fields [5]:

$$\partial_0 = \frac{\partial}{\partial t},\ \ \partial_i = \frac{\partial}{\partial x_i}\ (i = 1, 2, \cdots, n), \tag{2.1}$$