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AN EFFICIENT PARALLEL PROCESSING OPTIMAL CONTROL SCHEME FOR A CLASS OF NONLINEAR COMPOSITE SYSTEMS*

A. JAJARMI†
Department of Electrical Engineering, University of Bojnord, P.O. Box 94531-1339, Bojnord, Iran
E-mail: a.jajarmi@ub.ac.ir

M. HAJIPOUR
Department of Mathematics, Faculty of Basic Sciences, Sahand University of Technology, P.O. Box 51335-1996, Tabriz, Iran
E-mail: hajipour@sut.ac.ir

Abstract This article presents an efficient parallel processing approach for solving the optimal control problem of nonlinear composite systems. In this approach, the original high-order coupled nonlinear two-point boundary value problem (TPBVP) derived from the Pontryagin's maximum principle is first transformed into a sequence of lower-order decoupled linear time-invariant TPBVPs. Then, an optimal control law which consists of both feedback and forward terms is achieved by using the modal series method for the derived sequence. The feedback term specified by local states of each subsystem is determined by solving a matrix Riccati differential equation. The forward term for each subsystem derived from its local information is an infinite sum of adjoint vectors. The convergence analysis and parallel processing capability of the proposed approach are also provided. To achieve an accurate feedforward-feedback suboptimal control, we apply a fast iterative algorithm with low computational effort. Finally, some comparative results are included to illustrate the effectiveness of the proposed approach.

Key words Modal series method; nonlinear composite system; optimal control; parallel processing

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1 Introduction

In general, a composite system, also called large-scale or interconnected system, is considered as a dynamic system which is composed of some lower-order interconnected subsystems. Nowadays, these systems appear in a wide range of applications such as electric power systems, manufacturing processes, transportation networks, economic systems, chemical reactors, etc. Nevertheless, to design and control such processes using standard techniques severe limitations

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†Corresponding author

will quickly be appeared [1]. The standard control techniques impose a substantial computational burden and are typically sensitive to failures and modeling errors. Also, the most optimal control approaches cannot handle high dimensional systems with a large number of inputs and outputs. In order for any optimal control technique to be successful, the system structure must be exploited to obtain tractable algorithms. Therefore, many researchers look for some efficient strategies to successfully control the interconnected systems, theoretically and practically. In this study, we try to design an efficient optimal control technique to minimize the following finite-time quadratic cost functional

$$J=\frac{1}{2}\sum_{i=1}^{N}\left\{\int_{t_0}^{t_f}\left(x_i^T(t)\boldsymbol{Q_i}x_i(t)+u_i^T(t)\boldsymbol{R_i}u_i(t)\right)\mathrm{d}t\right\}, \tag{1.1}$$

subject to a nonlinear similar composite system which is decomposed into N interconnected subsystems as

$$\begin{cases}\dot{x}_i(t)=\boldsymbol{A_i}x_i(t)+\boldsymbol{B_i}u_i(t)+F_i(x(t)), \quad t_0<t<t_f,\\ x_i(t_0)=x_{i_0},\ x_i(t_f)=x_{i_f}, \quad i=1,2,\cdots,N,\end{cases} \tag{1.2}$$

where $x_i(t)\in\mathbb{R}^{n_i}$ and $u_i(t)\in\mathbb{R}^{m_i}$ are the state and control vectors of the i-th subsystem, respectively. The vector $x(t)=\left(x_1^T(t),x_2^T(t),\cdots,x_N^T(t)\right)^T$ is the state vector of the global system and $\sum\limits_{i=1}^{N}n_i=n$. The parameters x_{i_0} and x_{i_f} are the specified initial and final state vectors of the i-th subsystem, respectively. $F_i:\mathbb{R}^n\to\mathbb{R}^{n_i}$ is a nonlinear analytic vector field, where $F_i(0)=0$. Also, $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ are real constant matrices of appropriate dimensions and the pair $(\boldsymbol{A_i},\boldsymbol{B_i})$ is assumed to be completely controllable. The matrices $\boldsymbol{Q_i}\in\mathbb{R}^{n_i\times n_i}$ and $\boldsymbol{R_i}\in\mathbb{R}^{m_i\times m_i}$ are positive semi-definite and positive definite matrices, respectively.

For the nonlinear optimal control problem (OCP) given by Equations (1.1)–(1.2), the Pontryagin's maximum principle provides the following necessary conditions of optimality for $i=1,2,\cdots,N$

$$\begin{cases}\dot{x}_i(t)=\boldsymbol{A_i}x_i(t)-\boldsymbol{B_i}\boldsymbol{R_i}^{-1}\boldsymbol{B_i}^T\lambda_i(t)+F_i(x(t)), \quad t_0<t<t_f,\\ \dot{\lambda}_i(t)=-\boldsymbol{Q_i}x_i(t)-\boldsymbol{A_i}^T\lambda_i(t)+G_i(x(t),\lambda(t)), \quad t_0<t<t_f,\\ x_i(t_0)=x_{i_0},\ x_i(t_f)=x_{i_f},\end{cases} \tag{1.3}$$

where $\lambda_i(t)\in\mathbb{R}^{n_i}$ and $\lambda(t)=\left(\lambda_1^T(t),\lambda_2^T(t),\cdots,\lambda_N^T(t)\right)^T$ are the co-state vectors of the i-th subsystem and the global system, respectively. The function $G_i(x(t),\lambda(t))$ is expressed by

$$G_i(x(t),\lambda(t))\triangleq-\sum_{j=1}^{N}\left(\frac{\partial F_j(x(t))}{\partial x_i(t)}\right)^T\lambda_j(t). \tag{1.4}$$

Also, the optimal control law of the i-th subsystem is

$$u_i^*(t)=-\boldsymbol{R_i}^{-1}\boldsymbol{B_i}^T\lambda_i(t), \quad t_0<t<t_f. \tag{1.5}$$

The problem given by Equation (1.3) is a high-order coupled nonlinear two-point boundary value problem (TPBVP), which is decomposed into N lower-order interconnected sub-problems. This problem cannot be solved analytically or even numerically except in very special cases. During the past decades, various optimal control strategies have been constructed to overcome these difficulties. Among these strategies, the decentralized control [2], hierarchical control [3], successive approximation approach (SAA) [4], and the modal series method [5] have been interested