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ON SUBSONIC AND SUBSONIC-SONIC FLOWS IN THE INFINITY LONG NOZZLE WITH GENERAL CONSERVATIVES FORCE*

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Abstract In this article, we study irrotational subsonic and subsonic-sonic flows with general conservative forces in the infinity long nozzle. For the subsonic case, the varified Bernoulli law leads a modified cut-off system. Because of the local average estimate, conservative forces do not need any decay condition. Afterwards, the subsonic-sonic limit solutions are constructed by taking the extract subsonic solutions as the approximate sequences.

Key words Steady flow; homentropic; irrotation; subsonic flow; subsonic-sonic limit

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1 Introduction

Here, we consider the steady homentropic Euler equations with extract forces, which are written as:

$$\begin{cases} \mathrm{div}(\rho u) = 0, \\ \mathrm{div}(\rho u \otimes u) + \nabla p = \rho F, \\ \mathrm{curl} u = 0, \end{cases} \tag{1.1}$$

where $x = (x_1, \cdots, x_n) \in \mathbb{R}^n, n \geq 3$. $u = (u_1, \cdots, u_n) \in \mathbb{R}^n$ is the fluid velocity, while ρ, p, and F represent density, pressure, and extra forces, respectively. For the hometropic flow, the pressure p is a function of the density ρ, which is written as: $p = p(\rho)$. As usual, we require

$$p'(\rho) > 0, \quad 2p'(\rho) + \rho p''(\rho) > 0 \qquad \text{for } \rho > 0, \tag{1.2}$$

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which include the γ-laws flow with $p = \kappa\rho^\gamma$, for $\gamma > 1$ and $\kappa > 0$, and the isothermal flows, with $p = \kappa\rho$; see [11]. The Mach number is a non-dimensional ratio of the fluid velocity to the local sound speed,

$$M = \frac{|u|}{c},$$

where

$$|u| := \Big(\sum_{i=1}^{n} u_i^2\Big)^{1/2}$$

is the flow speed and

$$c = \sqrt{p'(\rho)}$$

is the local sound speed. The flow is subsonic when $M < 1$, while $M = 1$ means that the flow is locally sonic. Otherwise, $M > 1$ implies that the flow is supersonic.

Through this article, we consider the case where the extra force F is conservative, which means there exists a potential function ψ such that $F = \nabla\psi$. There are many nature and important examples on this type of forces in reality, for instance, the gravity field. Another usual example is the electric field.

One of classical problems on the steady compressible flows is the infinity long nozzle problem. Let $\Omega \subset \mathbb{R}^n$ be an infinitely long nozzle, which is a homomorphism on the unit cylinder $\mathbf{C} = B(0,1) \times \mathbb{R}$ in $\mathbb{R}^n$. The compressible fluid fills in the region Ω. At the $\partial\Omega$ boundary, the flow satisfies the slip condition:

$$u \cdot \nu = 0 \qquad \text{on } \partial\Omega, \tag{1.3}$$

where ν is the unit outward normal to the region Ω. One can derive the fixed mass flux property from $(1.1)_1$ and (1.3):

$$\int_{S_0} l \cdot \rho u \, \mathrm{d}s =: m,$$

where S_0 is an arbitrary cross section of the nozzle, and l is the unit outer normal of the domain S_0.

The theory of global subsonic flow in a variable nozzle was formulated by Bers in [4] in 1958. Then, the first rigorous proof of nozzle problem was achieved by Xie and Xin [27] by introducing the flux potential. Later, they extended it to the 3D axis-asymmetric nozzles case in [28]. The theorem for general infinitely long nozzle in $\mathbb{R}^n$ $(n \geq 2)$ was completed in Du-Xin-Yan [14]. The largely open nozzle case was proved by Liu and Yuan [23]. Besides the theory of nozzle problems, we also want to mention the study on another classical case: the airfoil problem. Frankl-Keldysh [24], Shiffman [25, 26], Bers [1–3], and Finn-Gilbarg [18] considered the irrotational, two-dimensional subsonic flow. Finn and Gilbarg [19] obtained the first result for higher dimensional subsonic flow past an obstacle under some restrictions on the Mach number. Then, for the three and higher dimensional case, Dong [12] and Dong-Ou [13] concluded these results to the situations where maximum Mach number is below 1. The respective case with extra conservative force is considered in [21]. For the rotational subsonic flows case, please refer to [5–7, 15–17, 29] .

On the basis of the the existence of exact subsonic solutions, one could consider the subsonic-sonic limit solutions by the compactness method. The compactness framework on sonic-subsonic irrotational flows in two dimension was introduced by [8, 27] independently. In [8], the general