Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2017,**37B**(3):806–835
Acta Mathematica Scientia 数学物理学报
http://actams.wipm.ac.cn

LARGE TIME BEHAVIOR OF SOLUTIONS TO 1-DIMENSIONAL BIPOLAR QUANTUM HYDRODYNAMIC MODEL FOR SEMICONDUCTORS*

Xing LI (李杏)
College of Mathematics and Statistics, Shenzhen University, Shenzhen 518060, China
E-mail: *lixing@szu.edu.cn*

Yan YONG (雍燕)
College of Science, University of Shanghai for Science and Technology, Shanghai 200093, China
E-mail: *yongyan@usst.edu.cn*

Abstract In this article, we study the 1-dimensional bipolar quantum hydrodynamic model for semiconductors in the form of Euler-Poisson equations, which contains dispersive terms with third order derivations. We deal with this kind of model in one dimensional case for general perturbations by constructing some correction functions to delete the gaps between the original solutions and the diffusion waves in L^2-space, and by using a key inequality we prove the stability of diffusion waves. As the same time, the convergence rates are also obtained.

Key words Bipolar quantum hydrodynamic; diffusion waves; semiconductor; Euler-Poisson euqations; asymptotic behavior

2010 MR Subject Classification 35A01; 35A02; 39A14

1 Introduction

In this article, we study the isentropic Euler-Poisson equations for the bipolar quantum hydrodynamical model of semiconductor device,

$$\begin{cases} n_{1t} + J_{1x} = 0, \\ J_{1t} + \Big(\dfrac{J_1^2}{n_1} + p(n_1)\Big)_x = n_1 E + \dfrac{\eta^2}{2} n_1 \Big(\dfrac{(\sqrt{n_1})_{xx}}{\sqrt{n_1}}\Big)_x - \dfrac{J_1}{\tau_1}, \\ n_{2t} + J_{2x} = 0, \\ J_{2t} + \Big(\dfrac{J_2^2}{n_2} + p(n_2)\Big)_x = -n_2 E + \dfrac{\eta^2}{2} n_2 \Big(\dfrac{(\sqrt{n_2})_{xx}}{\sqrt{n_2}}\Big)_x - \dfrac{J_2}{\tau_2}, \\ E_x = n_1 - n_2, \end{cases} \tag{1.1}$$

*Received February 29, 2016. X. Li's research was supported in part by NSFC (11301344); Y. Yong's research was supported in part by NSFC (11201301).

for $x \in \mathbb{R},\ t \in \mathbb{R}^+$. Here, n_1, n_2, J_1, J_2, and E represent the electron density, the hole density, the electron velocity, the hole velocity, and the electric field, respectively. τ_1 and τ_2 denote the relaxation time of electron and hole, respectively. As we are interested in the large time behavior, we assume $\tau_1 = \tau_2 = \tau \leq 1$. The nonlinear functions $p(s)$ denote the pressures of the electrons and the holes, respectively, which are smooth, strictly increasing, and nonnegative, that is,

$$p(s) \geq 0, \quad p'(s) > 0 \ \text{ for } s > 0. \tag{1.2}$$

Quantum hydrodynamic models of semiconductor equations was investigated by many people for both unipolar and bipolar situations. Especially, for unipolar hydrodynamic models, there exist rich results, such as well-posedness of steady-state solutions in [4, 7, 8], and their stability in [11, 14, 21, 26, 30, 44], the golbal existence of classical and/or the entropy weak solutions in [2, 27, 34, 45, 49], the large time behavior of solutions in [14, 26, 28], and the zero relaxation limit problems in [1, 10, 15], and so on. However, the study for bipolar hydrodynamic semiconductor equations is quite limited. In 1-D case, Natalini [40], and Hsiao and Zhang [15, 16] established the global entropic weak solutions in the framework of compensated compactness on the whole real line and spatial bounded domain, respectively. Hattori and Zhu [51] proved the stability of steady-state solutions for a recombined bipolar hydrodynamic model. Gasser, Hsiao, and H. Li [9], and Huang and Y. Li [18] investigated the large time behavior of both small smooth and weak solutions, respectively. Furthermore, Y. Li [29] studied the relaxation limit of a bipolar isentropic hydrodynamic models for semiconductors with small momentum relaxation time. In n-D case, F. Huang, M. Mei, and Y. Wang [21] proved the stability of planar diffusion waves recently.

Physically, the frictional damping usually causes the dynamical system to possess the nonlinear diffusive phenomena. Such interesting phenomena for 1-D compressible Euler equations with damping was investigated firstly by Hsiao and Liu [12]. Since then, this problem has attracted considerable attention; for example, see [21, 33, 38, 41–43, 48, 50] and the references therein.

For system (1.1) without the nonlinear dispersive terms of third order derivations, Gasser, Hsiao, and H. Li [9] and Huang, and Y. Li [18] investigated the large time behavior of both small smooth and weak solutions when the difference between the initial electron mass and the initial hole mass is zero. Later, removing the need of the difference between the initial electron mass and the initial hole mass to be zero, F. Huang, M. Mei, and Y. Wang [21] proved the stability of planar diffusion waves in n-D case. In this article, our main interesting is to investigate the large time behavior of the Euler-Poisson equations (1.1) when the initial electron mass and the initial hole mass are nonzero, which contains dispersive terms with third order derivations. Under this situation, there exist some particular difficults. First, the correction functions used in [9] cannot be applied anymore because of the effect of the electric field. Second, the strategy of antiderivative used in [9, 21] (in 1-D case) cannot be directly used in our case because of the appearance of the high order terms. To overcome these difficulties, we first apply the correction functions used in [21], then establish a basic estimate, and later combine the antiderivative strategy with a technical inequality, which was contributed by Huang, Li, and Matsumura [17], to obtain the energy estimates.