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ON COLLISION LOCAL TIME OF TWO INDEPENDENT FRACTIONAL ORNSTEIN-UHLENBECK PROCESSES*

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Abstract In this article, we study the existence of collision local time of two independent d-dimensional fractional Ornstein-Uhlenbeck processes $X_t^{H_1}$ and $\widetilde{X}_t^{H_2}$, with different parameters $H_i \in (0,1), i=1,2$. Under the canonical framework of white noise analysis, we characterize the collision local time as a Hida distribution and obtain its' chaos expansion.

Key words Collision local time; fractional Ornstein-Uhlenbeck processes; generalized white noise functionals; choas expansion

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1 Introduction

A diffusion process $X = \{X_t, t \geq 0\}$ starting from $x \in \mathbb{R}$ is called an Ornstein-Uhlenbeck (O-U) process, which can be constructed as the unique strong solution of Itô stochastic differential equation

$$\mathrm{d}X_t = -X_t \mathrm{d}t + v \mathrm{d}B_t, \quad X_0 = x, \tag{1.1}$$

where $\sigma > 0$ and $B = \{B_t, t \geq 0\}$ is a Brownian motion. At first, Ornstein-Uhlenbeck process describes the velocity of massive Brownian particles under the influence of friction. The process is stationary, Gaussian, and Markovian, and has the significant mean-reverting property. So, Ornstein-Uhlenbeck processes are extensively used in many fields such as finance and physical sciences.

In recent years, there has been a growing interest to the study of fractional Brownian motion (fBm), due to its interesting properties and applications in various scientific areas such

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as telecommunications, mechanics, and finance (see [2] and the reference therein). Some authors studied the local times of fractional Brownian motions, such as the works in [9, 10, 17] for the intersection case and in [6, 16] for the collision case. In particular, Oliveira et al [10] considered the intersection local times of independent fractional Brownian motions as generalized white noise functionals. Hence, the problems on O-U processes driven by fractional Brownian motions, that is fractional O-U processes, had given rise to many concerns. For examples, Shen et al in [13] investigated the Hölder regularity of the local times of fractional O-U processes using classical method. Yan et al in [15] proved Tanaka formula for fractional O-U processes and some properties of local times. And the problems of estimating the parameters of fractional O-U processes were discussed in [5, 8, 11, 12].

The purpose of this article is to study the existence of collision local times of fractional O-U processes. Let $X_t^{H_1}$ and $\widetilde{X}_t^{H_2}$ be two independent fractional O-U processes. The collision local time of fractional O-U processes is formally defined as

$$L_H(X^{H_1}, \widetilde{X}^{H_2}) = \int_0^T \delta(X_t^{H_1} - \widetilde{X}_t^{H_2})\mathrm{d}t, \tag{1.2}$$

where $\delta(x)$ is a Dirac delta function and $T > 0$. The intuitive idea of the local time $L_H(X^{H_1}, \widetilde{X}^{H_2})$ of processes $X_t^{H_1}$ and $\widetilde{X}_t^{H_2}$ is that $L_H(X^{H_1}, \widetilde{X}^{H_2})$ characterizes collision time during the interval $[0, T]$. On account of the complicated structure of fractional O-U processes, there is little attention on collision local time of these processes by using white noise analysis theory. Meanwhile, because these processes are not semimartingales (or Markovians), many classical methods in stochastic analysis can not be used to deal with these problems. Motivated by [10, 15], we consider a continuous version of fractional O-U processes and study the existence of the collision local time of two independent fractional O-U processes with different parameters in $(0, 1)$. We prove that the collision time is a Hida distribution and belong to $L^2(\Omega, \mathcal{F}, P)$ by using white noise analysis.

This article is organized as follows. In Section 2, we give the definition of fBm and provide some background about white noise analysis. In Section 3, we present the main results and their proofs.

2 Preliminaries

In this section, we first introduce the definition of fractional Brownian motion, and briefly recall some notions and facts in white noise analysis (for more details, we refer to [2, 10]).

2.1 Fractional Brownian motion

A fractional Brownian motion $B^H = \{B_t^H, t \geq 0\}$ on $\mathbb{R}^d$ with Hurst parameter $H \in (0, 1)$ is a d-dimensional Gaussian process with covariance

$$E[B_t^{H,i} B_s^{H,j}] = \frac{\delta_{i,j}}{2}(t^{2H} + s^{2H} - | t - s |^{2H}),$$

where $i, j = 1, 2, \cdots, d$, and $s, t \geq 0$. For $H = \frac{1}{2}$, the fractional Brownian motion is reduced to Brownian motion, which means that fBm is a generalized Gaussian process. And, we assume that the B^H is defined on a complete probability space $(\Omega, \mathcal{F}, P)$, where the filtration $\mathcal{F}$ is generated by B^H.