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GLOBAL REGULARITY TO THE 2D INCOMPRESSIBLE MHD WITH MIXED PARTIAL DISSIPATION AND MAGNETIC DIFFUSION IN A BOUNDED DOMAIN*

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Abstract This article considers the global regularity to the initial–boundary value problem for the 2D incompressible MHD with mixed partial dissipation and magnetic diffusion. To overcome the difficulty caused by the vanishing viscosities, we first establish the elliptic system for u_x and b_y, which are estimated by $\nabla \times u_x$ and $\nabla \times b_y$, respectively. Then, we establish the global estimates for $\nabla \times u$ and $\nabla \times b$.

Key words Global classical solution; incompressible MHD; initial–boundary value problem; partial dissipation

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1 Introduction

Magnetohydrodynamics (MHD) is the study of the interaction between magnetic fields and moving conducting fluids, which can be described by the following system in $\mathbb{R}^2$:

$$\begin{cases} u_t + u \cdot \nabla u + \nabla \pi = \mu_1 u_{xx} + \mu_2 u_{yy} + b \cdot \nabla b, \\ b_t + u \cdot \nabla b = \nu_1 b_{xx} + \nu_2 b_{yy} + b \cdot \nabla u, \\ \text{div} u = 0, \quad \text{div} b = 0, \end{cases} \tag{1.1}$$

where $(x, y) \in \mathbb{R}^2$, $t \geq 0$. The unknown functions $u = (u^1, u^2)$, $b = (b^1, b^2)$ and π are the velocity field, magnetic field, and pressure, respectively. The constants μ_1, μ_2, ν_1, and ν_2 are nonnegative real parameters.

We study global solutions to (1.1) with initial data:

$$u(x, y, 0) = u_0, \quad b(x, y, 0) = b_0, \quad (x, y) \in \Omega, \tag{1.2}$$

where

$$\Omega = [0, 1] \times [0, 1],$$

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and boundary condition:

$$\begin{cases} u^1 = \partial_x u^2 = 0, & \text{as } x = 0, 1, \\ u^2 = \partial_y u^1 = 0, & \text{as } y = 0, 1, \\ b^1 = \partial_x b^2 = 0, & \text{as } x = 0, 1, \\ b^2 = \partial_y b^1 = 0, & \text{as } y = 0, 1. \end{cases} \tag{1.3}$$

Let us first briefly review some existence theories on the study of the incompressible MHD equations. For the homogenous MHD system with full dissipation and magnetic diffusion, Duraut-Lions [6] constructed a class of global weak solution with finite energy and a class of local strong solution. Local well-posedness was established by Sermange-Temam [10] for any initial datum in $H^s(\mathbb{R}^3)$ with $s \geq 3$. In particular, the 2D local solution was proved to be global. For the case when magnetic diffusion is zero, Bardos-Sulem-Sulem [2] proved the global existence of the classical solution when the initial data (u_0, b_0) close to the equilibrium state $(0, B_0)$. Lin-Xu-Zhang [4] and Xu-Zhang [9] established the global wellposedness in two and three dimensional space, respectively, under the assumption that the initial datum are sufficiently close to the equilibrium state. The global existence of smooth solutions was proved by Lei [15] for the ideal MHD with axially symmetric initial datum in $H^s(\mathbb{R}^3)$ with $s \geq 2$. Recently, Cao-Wu [3] obtained the global classical solutions with mixed partial dissipation and magnetic diffusion in $\mathbb{R}^2$. And Cao-Wu's result was generalized to the 3D case by Wang-Wang [5] under the assumption that the H^1-norm of initial data is small.

For the nonhomogenous case, Desjardins-Le Bris [1], Gerbeau-Le Bris [8] studied the global existence of weak solutions with finite energy in the whole space or in a torus. Global existence of strong solutions with small initial data in some Besov spaces was considered by Abidi-Paicu [7]. Chen-Tan-Wang [11] extended the local existence in presence of vacuum. Recently, global existence of strong solutions with vacuum in the two dimensional space is proved by Huang-Wang [13]. In three dimensional space, Li-Wang [14] obtained the global strong solutions with small initial data and no vacuum in bounded domains.

The results in [3–5, 9, 15] are established in the whole two or three–dimensional space. A natural question is whether those results can be generalized to bounded domains. In this article, we shall prove the global regularity to the initial–boundary value problem (1.1)–(1.3) with mixed partial dissipation and magnetic diffusion.

Before stating the main results, we explain the notations used throughout this article. We denote

$$\begin{cases} \displaystyle\int f \triangleq \int_0^1 \int_0^1 f \mathrm{d}x\mathrm{d}y, \quad |f|_{L_x^2}^2 \triangleq \int_0^1 |f|^2 \mathrm{d}x, \\ \displaystyle\|f\|^2 = |f|_{L_{xy}^2}^2 \triangleq \int_0^1 \int_0^1 |f|^2 \mathrm{d}x\mathrm{d}y, \quad \omega \triangleq \nabla \times u, \quad j \triangleq \nabla \times b. \end{cases}$$

The main result in this article reads as follows.

Theorem 1.1 Let $\mu_1 = \nu_2 = 1$, and $\nu_1 = \mu_2 = 0$. Assume that

$$u_0 \in H^2, \quad b_0 \in H^2, \quad \mathrm{div} u_0 = 0, \quad \mathrm{div} b_0 = 0, \tag{1.4}$$