Available online at www.sciencedirect.com
ScienceDirect
Acta Mathematica Scientia 2017,**37B**(2):511–526
Mathematica Acta Scientia 数学物理学报
http://actams.wipm.ac.cn

NONLINEAR ANALYSIS ON THE VIBRATION OF ELASTIC PLATES*

Min DING (丁敏)
Department of Mathematics, School of Science, Wuhan University of Technology, Wuhan 430070, China
E-mail: *minding@whut.edu.cn*

Shengbo GONG (龚胜波)
School of Mathematical Science and School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
E-mail: *jiaoda20064154@sjtu.edu.cn*

Abstract We consider the vibration of elastic thin plates under certain reasonable assumptions. We derive the nonlinear equations for this model by the Hamilton Principle. Under the conditions on the hyperbolicity for the initial data, we establish the local time well-posedness for the initial and boundary value problem by Picard iteration scheme, and obtain the estimates for the solutions.

Key words Vibration of elastic plates; Hamilton principle; well-posedness; Picard iteration

2010 MR Subject Classification 35A01; 35L20; 35B35

1 Introduction

In this article, we are concerned with the vibrations of elastic thin plates. This problem has been widely studied from the last century. In particular, there are a lot of works including theoretical analysis and numerical simulation on gyro linear models, and most of them are confirmed to be effective. However, some phenomenons can not be explained by these models, such as the self-excited vibration, harmonic resonance, ultra-harmonic resonance and so on. Recently, many systematic investigations were developed on the nonlinear analysis of the vibration. Wickert [7, 8] initially derived the equations for axial vibration by the Hamilton principle, and also adapted the method of KBM to discuss the nonlinear stability and bifurcation theory of the vibration for girder motions. Meanwhile, the method of multi-scaling was used by Riedel in [4] to analyze the asymptotic behavior of the axial vibration. Chen [1, 2] employed the incremental harmonic balance method (IHB) to explore the characteristic of nonlinear dynamics; the further references can be found in [1–3] and the references therein.

*Received March 14, 2016; revised May 23, 2016. Min Ding was supported in part by Innovation Award by Wuhan University of Technology under a project Grant 20410771; Shengbo Gong was supported in part by China Scholarship Council under Grant 201306230035.

Our motivation is to take both lateral and longitudinal vibrations of the motion mechanism of elastic thin plates into consideration. By the Hamilton variation principle, the system can be formulated by

$$\frac{\partial^2 U}{\partial t^2} - A_1(U)\frac{\partial^2 U}{\partial x_1^2} - A_2(U)\frac{\partial^2 U}{\partial x_2^2} - A_3(U)\frac{\partial^2 U}{\partial x_1 \partial x_2} = 0, \quad t > 0,\ x = (x_1, x_2) \in \Omega, \tag{1.1}$$

where $U = (u, v, w)$, u, v, and w stand for the horizontal, vertical and longitudinal replacements of the plates, respectively. $A_i(U)$ $(i = 1, 2, 3)$ is the symmetric matrix defined later. Ω is a bounded domain with smooth boundary in $\mathbb{R}^2$.

The initial and boundary data are given by

$$\begin{cases} U|_{t=0} = U_0(x_1, x_2), \qquad U_t|_{t=0} = U_1(x_1, x_2), \\ U|_{\partial\Omega} = g(t, x_1, x_2). \end{cases} \tag{1.2}$$

Note the fact that almost all the physical vibrations have finite speed of propagation, which implies that equations (1.1) should be hyperbolic at least in the state space. Actually, we will confirm it under the following assumption:

(A1) Assume that $\nu \in (0, 1)$ is Poisson's ratio. Denote $d = \frac{1+\nu}{1-\nu}$ and $\lambda = \frac{d-1}{2(d+1)}$. Suppose that (u, v, w) satisfies the inequalities

$$|u_{x_1}| \le \lambda (w_{x_1})^2, \quad |v_{x_2}| \le \lambda (w_{x_2})^2, \quad |u_{x_2}| + |v_{x_1}| \le \lambda |w_{x_1}||w_{x_2}|, \text{ and } |w_{x_1} w_{x_2}| \ne 0.$$

Then, the main results of this article can be given by

Theorem 1.1 Assume that the initial and boundary data are s-th compatibility for some integer $s > 3$, and that the hyperbolic condition (A1) holds for U_0 and U_1, where

$$U_0 \in H^{s+1},\ \ U_1 \in H^s \ \text{ and } \ g \in H^{s+1}([0, T];\ \partial\Omega).$$

Then, there exists a positive constant $T > 0$ depending on the initial data such that (1.1)–(1.2) admits a unique solution U satisfying

$$||U||_{L^\infty([0,T];\ H^s)} + ||U_t||_{L^\infty([0,T];\ H^{s-1})} \le C(||U_0||_{H^{s+1}} + ||U_1||_{H^s} + \|g\|_{H^{s+1}([0,T];\ \partial\Omega)}), \tag{1.3}$$

where C is a positive constant, depending on $\|U_0\|_{H^{s+1}}$, $\|U_1\|_{H^s}$, and $\|g\|_{H^{s+1}([0,T];\ \partial\Omega))}$.

This article can be organized as follows. In Section 2, we derive the equations for the vibration of the thin plates by Hamilton principle. Section 3 is contributed to constructing the approximate solution by Picard iteration scheme and concludes this article with the proof of Theorem 1.1.

2 The Derivation of Equations for the Vibration of Thin Plates

In this section, we apply the Hamilton principle to derive the mathematical model for the vibration of thin plates. For simplicity, we will consider it under the the following assumptions:

(A2) The Kirchhoff thickness of the thin plates is negligible, and the area density is a positive constant, and without loss of generality, let $\rho = 1$.

(A3) The plates have very well-flexible scalability, and the bending rigidity is not neglected.

(A4) The vibration of the plates satisfies the Kirchhoff theory, which means that the longitudinal vibration is still perpendicular to the plates.