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CONTROLLABILITY OF NEUTRAL STOCHASTIC EVOLUTION EQUATIONS DRIVEN BY FRACTIONAL BROWNIAN MOTION*

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Abstract In this paper, we investigate the controllability for neutral stochastic evolution equations driven by fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$ in a Hilbert space. We employ the α-norm in order to reflect the relationship between H and the fractional power α. Sufficient conditions are established by using stochastic analysis theory and operator theory. An example is provided to illustrate the effectiveness of the proposed result.

Key words stochastic evolution equations; fractional Brownian motion; controllability

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1 Introduction

The focus of this investigation is the controllability problem for a class of neutral stochastic evolution equations of the following form

$$\begin{cases} \mathrm{d}[x(t) - g(t, x(t))] = \Big[Ax(t) + Lu_x(t) + f(t, \int_0^t h(s, x(s))\mathrm{d}s)\Big]\mathrm{d}t \\ \qquad\qquad\qquad\qquad\quad + G\,\mathrm{d}B^H(t), \quad t \in [0, T], \\ x(0) = x_0 \end{cases} \tag{1.1}$$

in a real separable Hilbert space $\mathcal{H}$ with inner product $(\cdot,\cdot)$ and norm $||\cdot||$, where $A : D(A) \subset \mathcal{H} \to \mathcal{H}$ is the infinitesimal generator of a strong continuous semigroup of bounded linear operators $\{S(t); t \geq 0\}$. B^H is a cylindrical fractional Brownian motion defined on a filtered complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. The control function $u_x(\cdot)$ takes values in $L^2([0,T], U)$, the Hilbert space of admissible control functions for a separable Hilbert space U. L is a bounded

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linear operator from U to $\mathcal{H}$. For $0 < \alpha \leq 1$, denote by $D_\alpha := D(A^\alpha)$ the domain of the fractional power operator A^α, it is a Banach space with the norm $\|x\|_\alpha := A^\alpha x$ for $x \in D_\alpha$ (see [18]). $f : [0,T] \times D_\alpha \to \mathcal{H}$, $g : [0,T] \times D_\alpha \to \mathcal{H}$, $G \in L_2^0(V, \mathcal{H})$, $h : [0,T] \times D_\alpha \to D_\alpha$ are ppropriate functions specified later, $x_0 \in D_\alpha$ is an $\mathcal{F}_0$-measurable random variable independent of B^H with finite second moment.

It is known that fractional Brownian motion (fBm, for short) is a generalization of Brownian motion, it is a family of centered Gaussian processes with continuous sample paths that are indexed by the Hurst parameter $H \in (0,1)$. fBm admits the stationary increments, self-similarity and Hölder's continuity. Due to these compact properties and its applications in various scientific fields including turbulence, telecommunications, network traffic analysis and medicine, in recent years, there has been considerable interest in studying fBm, one refer to [5, 10, 12–14, 17, 20] and references therein for complete presentation of this subject.

On the other hand, many real world problems in science and engineering can be modeled by stochastic (partial) differential equations driven by fBm. In particular, many types of stochastic differential equations driven by fBm in infinite dimension received much attention, for example, Grecksch and Anh [2] proved the existence and uniqueness of the solution to a semilinear stochastic parabolic equation with fractional noise; Duncan et al. [1, 15] established the weak, strong and mild solutions to stochastic equations with multiplicative fractional noise; Boufoussi and Hajji [4] investigated the existence of neutral stochastic functional differential equations driven by fBm; Caraballo et al. [7] proved the existence and exponential behavior of mild solutions to stochastic delay evolution equations with fractional noise.

We know that the control problem for stochastic systems is one of the prevalent and fundamental concept in stochastic control theory. Controllability for stochastic systems driven by Brownian motion are well investigated, we refer to [9, 19] and references therein. But only few works appeared about the controllabilty for stochastic systems driven by fBm, more precisely, Chen [6] considered the approximate controllability for semilinear stochastic equations driven by fBm, Ahmed [3] studied the approximate controllability of impulsive neutral stochastic differential equations with fBm, Lakhel [16] concerned the controllability of neutral stochastic functional integro-differential equations driven by fBm. It is shown that the conditions are also right for Brownian motion. In particular, the present controllability result of neutral stochastic system driven by fBm does not reflect adequately the relationship between fBm and the power of the operator A, so it does not reflect the characteristic of fBm. Motivated by this facts, our current consideration is the controllability of neutral stochastic evolution systems (1.1) via α-norm, when $\alpha < H$, we present the sufficient conditions to insure the complete controllability of (1.1).

A brief outline of this paper is given. In Section 2, we present some basic notations and preliminaries on the stochastic integral with respect to fBm in Hilbert space; in Section 3, the controllability result of system (1.1) is investigated by means of Banach fixed theorem and operator theory. At last, an example is presented to illustrate the effectiveness of the main result.