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EXACT CONTROLLABILITY AND CONTINUOUS DEPENDENCE OF FRACTIONAL NEUTRAL INTEGRO-DIFFERENTIAL EQUATIONS WITH STATE-DEPENDENT DELAY*

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Abstract In the present paper, with the help of the resolvent operator and some analytic methods, the exact controllability and continuous dependence are investigated for a fractional neutral integro-differential equations with state-dependent delay. As an application, we also give one example to demonstrate our results.

Key words controllability; continuous dependence; fractional neutral integro-differential equations; state-dependent delay; resolvent operator

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1 Introduction

In the last two decades, fractional differential equations have gained importance and popularity, due to its wide range of applications in varied fields of sciences and engineering, such as in physics, chemistry, biology, finance ect., which are richer in problems in comparison with corresponding theory of classical differential equations. For details, see the monographs of Kilbas, Srivastava, and Trujillo [15], Miller and Ross [22] as well as Podlubny [25].

Control theory of fractional differential equation is emerging recently. Many authors investigated exact and approximate controllability of nonlinear fractional differential equations (see [4, 5, 16, 17, 27, 28]). In recent years, controllability of fractional differential with bounded and unbounded delay have generated considerable interest among researchers. For example, for bounded delay case, Kumar and Sukavanam [16] proved sufficient conditions for the approximate controllability of fractional order semilinear systems described in the form

$$\begin{cases} {}^cD_t^{\alpha}x(t) = Ax(t) + Bu(t) + f(t, x(t-h)), & t \in I = [0, T], \\ x(t) = \varphi(t), & t \in [-h, 0], \end{cases}$$

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where ${}^cD_t^\alpha$ is the Caputo derivative of order α, $\frac{1}{2} < \alpha < 1$. For infinite delay case, Mophou and N'Guérékata [23] paid attention to the semilinear differential system of fractional order of the form

$$\begin{cases} {}^cD_t^\alpha x(t) = Ax(t) + Bu(t) + f(t, x_t), & t \in I = [0, T], \\ x(t) = \varphi(t), & t \in (-\infty, 0]. \end{cases}$$

They proved that the system is controllable when A generates an α-resolvent family $(\mathcal{S}_\alpha(t))_{t\geq 0}$ on a complex Banach space X and the control $u \in L^2([0, T]; X)$. For details, the readers can refer to the references (see [3, 6, 16, 26, 32, 33]).

Recently, the existence of fractional differential equations with state-dependent delay has received much attention, due to their important applications in mathematical models; see [1, 7, 12, 37] and the references therein. Das et al. [8] obtained the existence and uniqueness of mild solution of a second order stochastic neutral partial differential equation with state-dependent delay by use of measure of non-compactness. Particularly, in 2011, taking advantage of Leray-Schauder alternative fixed point theorem, Santos et al. [10] studied the existence of mild solutions for a class of abstract fractional neutral integro-differential equations with state-dependent delay. On the other hand, the controllability of the fractional differential equations or inclusions with state-dependent delay generated considerable interest among researchers. For example, Yan [35] gave the sufficient conditions for approximate controllability of fractional neutral integro-differential inclusions with state-dependent delay in Hilbert space by using the fixed point theorem of discontinuous multi-valued operators due to Dhage with the α-resolvent opertaor. Further, In Sakthivel and Ren [27] studied the approximate controllability for a class of fractional differential equations and control described in the abstract form

$$\begin{cases} {}^cD_t^\alpha x(t) = Ax(t) + Bu(t) + f(t, x_{\rho(t,x_t)}), & t \in I = [0, b], \\ x(t) = \phi \in P, \end{cases}$$

where $1 < \alpha < 2$; $A : D(A) \subset X \to X$ is a linear densely defined operator of sectorial type on a Banach space X. Very recently, Yan and Jia [36] concerned with the approximate controllability of partial fractional neutral stochastic functional integro-differential inclusions with state-dependent delay. Using fractional calculus, stochastic analysis theory and the fixed point theorem for multi-valued mapping due to Dhage with the α-resolvent operator, a new set of sufficient conditions for approximate controllability was obtained. For other related references, we refer the readers to [31, 34, 39]. However, we think the mild solution defined in Theorem 3.1 in [27] was not suitable and should be represented by the form $\mathcal{S}_\alpha(t)\phi(0) + \int_0^t \mathcal{T}_\alpha(t-s)[Bu(s) + f(s, x_{\rho(s,x_s)})]\mathrm{d}s$, where $\mathcal{S}_\alpha(t)$ is the solution operator and $\mathcal{T}_\alpha(t) = J_t^{\alpha-1}\mathcal{S}_\alpha(t) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)}\mathcal{S}_\alpha(s)\mathrm{d}s$. Therefore, it is natural and necessary to discuss precisely how to deal with the fractional-order control system with state-dependent delay. In addition, to our knowledge, no one has studied the continuous dependence of the fractional-order control system with state-dependent delay. So in here there are two differences between our results and the references [31, 34–36, 39]. One is that, instead of Dhage's fixed point theorem mentioned in the above papers [35] and [36], by using Krasnoselskii's theorem, Leray-Schauder alternative theorem as well as contraction mapping principle, respectively, we concentrate on the exact controllability for the fractional-order control system with state-dependent delay. The other is that we further