

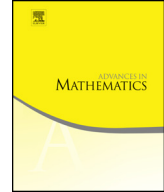


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Absolute continuity of non-homogeneous self-similar measures



Santiago Saglietti^{a,1}, Pablo Shmerkin^{b,2}, Boris Solomyak^{c,*,3}

^a Faculty of Industrial Engineering and Management, Technion, Haifa, 3200003, Israel

^b Department of Mathematics and Statistics, Torcuato di Tella University and CONICET, Av. Figueroa Alcorta 7350 (1425), Buenos Aires, Argentina

^c Department of Mathematics, Bar-Ilan University, Ramat-Gan, 5290002, Israel

ARTICLE INFO

Article history:

Received 1 October 2017

Received in revised form 2 May 2018

Accepted 19 June 2018

Available online xxxx

Communicated by Kenneth Falconer

MSC:

primary 28A78, 28A80

secondary 37A45, 42A38

Keywords:

Absolute continuity

Self-similar measures

Random measures

Convolutions

ABSTRACT

We prove that self-similar measures on the real line are absolutely continuous for almost all parameters in the super-critical region, in particular confirming a conjecture of S.-M. Ngai and Y. Wang. While recently there has been much progress in understanding absolute continuity for homogeneous self-similar measures, this is the first improvement over the classical transversality method in the general (non-homogeneous) case. In the course of the proof, we establish new results on the dimension and Fourier decay of a class of random self-similar measures.

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* Corresponding author.

E-mail addresses: saglietti.s@technion.ac.il (S. Saglietti), pshmerkin@utdt.edu (P. Shmerkin), bsolom3@gmail.com (B. Solomyak).

¹ S.S. was partially supported by Iniciativa Científica Milenio NC120062 and the Israel Science Foundation research grant 1723/14.

² P.S. was partially supported by Project PIP 11220150100355 (CONICET). Part of this work was completed while P.S. was visiting Mittag-Leffler institute as part of the program “Fractal Geometry and Dynamics”. P.S. thanks the organizers and staff for financial support and for a stimulating atmosphere.

³ B.S. was partially supported by the Israel Science Foundation grant 396/15.

1. Introduction and main results

1.1. Non-homogeneous self-similar measures

Given $\lambda_1, \lambda_2 \in (0, 1)$ and $p \in (0, 1)$, one can form the self-similar measure $\nu_{\lambda_1, \lambda_2}^p$ constructed with contraction ratios λ_1, λ_2 and weight p . That is, $\nu_{\lambda_1, \lambda_2}^p$ is the only Borel probability measure ν such that

$$\nu = p g_{1, \lambda_1} \nu + (1 - p) g_{2, \lambda_2} \nu,$$

where $g_{1, \lambda}(x) = \lambda x$ and $g_{2, \lambda}(x) = \lambda x + 1$. Here, and throughout the article, $g\nu$ denotes the image measure: $g\nu(A) = \nu(g^{-1}A)$ for all Borel sets A . When $\lambda_1 = \lambda_2$, the measure $\nu_{\lambda}^p := \nu_{\lambda, \lambda}^p$ is the classical (biased) Bernoulli convolution.

Write

$$s_{\lambda_1, \lambda_2}^p = \frac{p \log p + (1 - p) \log(1 - p)}{p \log \lambda_1 + (1 - p) \log \lambda_2}$$

for the similarity dimension of $\nu_{\lambda_1, \lambda_2}^p$. It is well known that if $s_{\lambda_1, \lambda_2}^p < 1$, then $\nu_{\lambda_1, \lambda_2}^p$ is always singular, even though if $\lambda_1 + \lambda_2 > 1$ its support is an interval.

In the case of Bernoulli convolutions, it was recently proved by the second author [14] that there exists an exceptional set $E \subset (1/2, 1)$ of zero Hausdorff dimension, such that ν_{λ}^p is absolutely continuous whenever $\lambda \in (1/2, 1) \setminus E$ and $s_{\lambda}^p := s_{\lambda, \lambda}^p > 1$ and, moreover, ν_{λ}^p has a density in L^q if $\lambda^{q-1} > p^q + (1-p)^q$ (this range is sharp up to the endpoint). See Theorem 1.3, Theorem 9.1 and discussion afterwards in [14] for details. This improved several earlier results [18, 12, 7, 13, 15] providing various bounds on the size of exceptional sets for different notions of smoothness of ν_{λ}^p . We remark, in particular, that in an early application of the so-called “transversality method”, the third author already in 1995 established absolute continuity of $\nu_{\lambda}^{1/2}$ for Lebesgue almost all $\lambda \in (1/2, 1)$ [18]. Also very recently, P. Varjú [20] established absolute continuity of ν_{λ}^p for many explicit algebraic values of λ near 1.

In the general case $\lambda_1 \neq \lambda_2$, much less is known. For $p = 1/2$, Jörg Neunhäuserer [10] and Sze-Man Ngai and Yang Wang [11] proved that $\nu_{\lambda_1, \lambda_2}^{1/2}$ is absolutely continuous for almost all λ_1, λ_2 in a certain simply connected region which is very far from covering the whole super-critical parameter region $\lambda_1 \lambda_2 > 1/4$ (which corresponds to $s_{\lambda_1, \lambda_2}^{1/2} > 1$), and in particular is disjoint from a neighborhood of $(1, 1)$. Ngai and Wang conjectured that, in fact, $\nu_{\lambda_1, \lambda_2}^{1/2}$ is absolutely continuous for almost all $\lambda_1, \lambda_2 \in (0, 1)$ such that $\lambda_1 \lambda_2 > 1/4$. This fits into the more general folklore conjecture that self-similar measures should be generically absolutely continuous in the super-critical regime. Recently, Hochman [8] proved that $\nu_{\lambda_1, \lambda_2}^p$ has Hausdorff dimension 1 for all λ_1, λ_2 such that $s_{\lambda_1, \lambda_2}^p > 1$, outside of an exceptional set of Hausdorff dimension 1, which is uniform in p . However, it does not seem possible to obtain absolute continuity from Hochman’s approach.

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