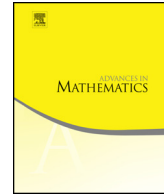




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Nonlocal curvature and topology of locally conformally flat manifolds



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ABSTRACT

In this paper, we focus on the geometry of compact conformally flat manifolds (M^n, g) with positive scalar curvature. Schoen–Yau proved that its universal cover (\tilde{M}^n, \tilde{g}) is conformally embedded in \mathbb{S}^n such that M^n is a Kleinian manifold. Moreover, the limit set of the Kleinian group has Hausdorff dimension $< \frac{n-2}{2}$. If additionally we assume that the non-local curvature $Q_{2\gamma} \geq 0$ for some $1 < \gamma < 2$, the Hausdorff dimension of the limit set is less than or equal to $\frac{n-2\gamma}{2}$. If $Q_{2\gamma} > 0$, then the above inequality is strict. Moreover, the above upper bound is sharp. As applications, we obtain some topological rigidity and classification theorems in lower dimensions.

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1. Introduction

Compact locally conformally flat manifolds with positive scalar curvature can be viewed as Kleinian manifolds by Schoen–Yau’s fundamental work in [17]. That is, if (M^n, g) is a compact locally conformally flat manifold with $R_g > 0$, then the universal cover $(\widetilde{M}^n, \tilde{g})$ can be conformally embedded in the standard sphere (\mathbb{S}^n, g_1) . Moreover, $\pi_1(M^n)$ is isomorphic to a Kleinian group $\Gamma \leq \text{Conf}(\mathbb{S}^n)$ such that

$$\widetilde{M}^n \cong \Omega(\Gamma) \equiv \mathbb{S}^n \setminus \Lambda, \quad (1.1)$$

where $\Lambda \equiv \Lambda(\Gamma)$ is the limit set of the Kleinian group Γ . In [17], Schoen–Yau also proved the following Hausdorff dimension estimate on the limit set Λ in the above setting,

$$\dim_{\mathcal{H}}(\Lambda) < \frac{n-2}{2}. \quad (1.2)$$

The above Hausdorff dimension estimate immediately gives homotopy vanishing and homology vanishing results, which are interesting topological obstructions for conformally flat manifolds with nonnegative scalar curvature (see [17] for more details).

In this paper, we will generalize the above theory to the fractional setting. In conformal geometry, scalar curvature R_g arises as the zeroth order term of the conformal Laplacian operator. More precisely, denote $J_g \equiv \frac{R_g}{2(n-1)}$, then

$$P_2 \equiv -\Delta_g + \frac{n-2}{2}J_g. \quad (1.3)$$

It is standard that the second order conformal Laplacian operator P_2 satisfies the following conformal covariance property: For $n \geq 3$, let $\hat{g} = v^{\frac{4}{n-2}}g$ and let \hat{P}_2 be the conformal Laplacian with respect to the conformal metric \hat{g} , then

$$\hat{P}_2(u) = v^{-\frac{n+2}{n-2}}P_2(uv). \quad (1.4)$$

The fourth order analogy of P_2 is called *Paneitz operator*, which is defined by

$$P_4(u) \equiv (-\Delta_g)^2u + \text{Div}_g(4A_g\langle \nabla_g u, e_j \rangle e_j - (n-2)J_g \nabla_g u) + \frac{n-4}{2}Q_4 \cdot u, \quad (1.5)$$

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