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Mellin transformation, propagation, and abelian duality spaces

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ABSTRACT

For arbitrary field coefficients \mathbb{K} , we show that \mathbb{K} -perverse sheaves on a complex affine torus satisfy the so-called *propagation package*, i.e., the generic vanishing property and the signed Euler characteristic property hold, and the corresponding cohomology jump loci satisfy the propagation property and codimension lower bound. The main ingredient used in the proof is Gabber–Loeser’s Mellin transformation functor for \mathbb{K} -constructible complexes on a complex affine torus, and the fact that it behaves well with respect to perverse sheaves.

As a concrete topological application of our sheaf-theoretic results, we study homological duality properties of complex algebraic varieties, via *abelian duality spaces*. We provide new obstructions on abelian duality spaces by showing that their cohomology jump loci satisfy a propagation package. This is then used to prove that complex abelian varieties are the only complex projective manifolds which are abelian duality spaces. We also construct new examples of abelian duality spaces. For example, we show that if a smooth quasi-projective variety X satisfies a certain Hodge-theoretic condition and it admits a proper semi-small map (e.g., a closed embedding or

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a finite map) to a complex affine torus, then X is an abelian duality space.

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1. Introduction

Cohomology jump loci of rank-one local systems provide a unifying framework for the study of a host of questions concerning homotopy types of complex algebraic varieties. In particular, they can be used to tackle Serre’s problem concerning groups which can be realized as fundamental groups of complex quasi-projective manifolds.

By the classical Albanese map construction (e.g., see [18]), cohomology jump loci of a complex quasi-projective manifold can be understood via cohomology jump loci of constructible complexes of sheaves (or, if the Albanese map is proper, of *perverse sheaves*) on a semi-abelian variety. This motivates our investigation of cohomology jump loci of such complexes, and in particular of cohomology jump loci of perverse sheaves. Throughout this paper, we restrict ourselves to the study of perverse sheaves on a complex affine torus, but we allow any field of coefficients. Specifically, we show that for any field \mathbb{K} , the cohomology jump loci of \mathbb{K} -perverse sheaves on a complex affine torus satisfy the so-called *propagation package*, i.e., a list of properties which provide new obstructions on the category of perverse sheaves on a complex affine torus. The present work parallels results of Gabber–Loeser in the ℓ -adic setting, see [14].

As a concrete topological application of our sheaf-theoretic results, we study homological duality properties of complex algebraic varieties, via *abelian duality spaces*. Abelian duality spaces have been recently introduced in [10], by analogy with the duality spaces of Bieri–Eckmann [3], and they are particularly useful for explaining (non-)vanishing

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