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Maximum principles for the fractional p-Laplacian and symmetry of solutions

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A B S T R A C T

In this paper, we consider nonlinear equations involving the fractional p-Laplacian

$$(-\triangle)_p^s u(x)) \equiv C_{n,sp} PV \int\limits_{\mathbb{R}^n} \frac{|u(x)-u(y)|^{p-2}[u(x)-u(y)]}{|x-y|^{n+sp}} dy$$

$$= f(x,u).$$

We prove a *maximum principle for anti-symmetric functions* and obtain other key ingredients for carrying on the method of moving planes, such as a variant of the Hopf Lemma – *a boundary estimate lemma* which plays the role of the *narrow region principle*. Then we establish radial symmetry and monotonicity for positive solutions to semilinear equations involving the fractional p-Laplacian in a unit ball and in the whole space. We believe that the methods developed here

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can be applied to a variety of problems involving nonlinear nonlocal operators.

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1. Introduction

In this paper, we consider nonlinear equations involving the fractional p-Laplacian

$$(-\triangle)_p^s u(x)) = f(x,u) \tag{1}$$

with

$$\begin{aligned}(-\triangle)_p^s u(x) &= C_{n,sp} \lim_{\epsilon \to 0} \int\limits_{\mathbb{R}^n \setminus B_\epsilon(x)} \frac{|u(x)-u(y)|^{p-2}[u(x)-u(y)]}{|x-y|^{n+sp}} dy \\ &= C_{n,sp} PV \int\limits_{\mathbb{R}^n} \frac{|u(x)-u(y)|^{p-2}[u(x)-u(y)]}{|x-y|^{n+sp}} dy,\end{aligned}$$

where PV stands for the Cauchy principal value.

In order the integral to make sense, we require that

$$u \in C^{1,1}_{loc} \cap L_{sp}$$

with

$$L_{sp} = \{u \in L^{p-1}_{loc} \mid \int\limits_{\mathbb{R}^n} \frac{|1+u(x)|^{p-1}}{1+|x|^{n+sp}} dx < \infty\}.$$

Interested readers may find the details in the proof of Lemma 5.2 in the Appendix.

In the special case when $p = 2$, $(-\triangle)_p^s$ becomes the well-known fractional Laplacian $(-\triangle)^s$. The nonlocal nature of these operators make them difficult to study. To circumvent this, Caffarelli and Silvestre [2] introduced the *extension method* which turns the nonlocal problem involving the fractional Laplacian into a local one in higher dimensions. This method has been applied successfully to study equations involving the fractional Laplacian, and a series of fruitful results have been obtained (see [1] [11] and the references therein). One can also use the *integral equations method*, such as *the method of moving planes in integral forms* and *regularity lifting* to investigate equations involving the fractional Laplacian by first showing that they are equivalent to the corresponding integral equations [9] [8] [4].

However, when working on the extended problems or the corresponding integral equations, sometimes one needs to impose extra conditions on the solutions, which would not

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