Advances in Mathematics 335 (2018) 838–877

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Classifications of exact structures and Cohen–Macaulay-finite algebras

Haruhisa Enomoto

Graduate School of Mathematics, Nagoya University, Chikusa-ku, Nagoya 464-8602, Japan

ARTICLE INFO

Article history:
Received 26 September 2017
Received in revised form 29 June 2018
Accepted 9 July 2018
Available online xxxx
Communicated by Henning Krause

MSC:
18E10
16G10
18E05

Keywords:
Exact category
Grothendieck group
CM-finite Iwanaga–Gorenstein algebra
Cotilting module

ABSTRACT

We give a classification of all exact structures on a given idempotent complete additive category. Using this, we investigate the structure of an exact category with finitely many indecomposables. We show that the relation of the Grothendieck group of such a category is generated by AR conflations. Moreover, we obtain an explicit classification of (1) Gorenstein-projective-finite Iwanaga–Gorenstein algebras, (2) Cohen–Macaulay-finite orders, and more generally, (3) cotilting modules U with ${}^{\perp}U$ of finite type. In the appendix, we develop the AR theory of exact categories over a noetherian complete local ring, and relate the existence of AR conflations to the AR duality and dualizing varieties.
© 2018 Elsevier Inc. All rights reserved.

Contents

E-mail address: m16009t@math.nagoya-u.ac.jp.

https://doi.org/10.1016/j.aim.2018.07.022
0001-8708/© 2018 Elsevier Inc. All rights reserved.

1. Introduction

In the representation theory of finite-dimensional algebras, one of the most important subjects is to *classify certain categories of finite type*. Here we say that an additive k-category over a field k is *of finite type* if it has only finitely many indecomposable objects up to isomorphism. The aim of this paper is to give a classification of exact categories of finite type, and thereby provide an explicit classification of all GP-finite Iwanaga–Gorenstein algebras. Let us explain the motivation for this.

First we recall how categories of finite type have been studied in the representation theory. It is well-known that an abelian Hom-finite k-category of finite type is nothing but the category $\mathsf{mod}\,\Lambda$ of finitely generated Λ-modules over some representation-finite k-algebra Λ (see [21, 8.2] or Proposition 3.14 below). A classification of such algebras is one of the main problems in the representation theory of algebras, and has been studied widely by a number of papers, e.g. [20,40,13,22]. For the case of representation-finite R-orders over a noetherian local ring R, we refer the reader to [2,18,25,35,38,44]. Besides abelian categories, triangulated categories of finite type also has been investigated, e.g. in [1]. Such triangulated categories naturally arise in the representation of algebras and in the categorification of cluster algebras.

Among other things, the observation by Auslander [4] is of particular importance to us when we deal with categories of finite type. Let $\mathcal{E}$ be a Hom-finite k-category of finite type and consider the algebra $\Gamma := \operatorname{End}_{\mathcal{E}}(M)$, where M is a direct sum of all non-isomorphic indecomposables in $\mathcal{E}$. This Γ is called an *Auslander algebra* of $\mathcal{E}$, and categorical properties of $\mathcal{E}$ should be related to homological properties of Γ. For example, the condition $\mathcal{E}$ being abelian is equivalent to a certain homological condition of Γ, that is, $\operatorname{gl.dim}\Gamma \leq 2 \leq \operatorname{dom.dim}\Gamma$. This is called the *Auslander correspondence*, and is now the basic and important viewpoint in the representation theory.

However, most of the algebras are representation-wild, and it is hopeless to understand the whole structure of the module category. Thus nice subcategories of module categories

Download English Version:

https://daneshyari.com/en/article/8904660

Download Persian Version:

https://daneshyari.com/article/8904660

Daneshyari.com