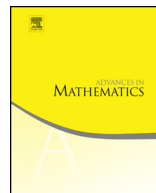




Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



The Beilinson regulator is a map of ring spectra

Ulrich Bunke^a, Thomas Nikolaus^b, Georg Tamme^{c,*}¹^a Fakultät für Mathematik, Universität Regensburg, 93040 Regensburg, Germany^b FB Mathematik und Informatik, Universität Münster, Einsteinstr. 62, 48149 Münster, Germany^c Fakultät für Mathematik, Universität Regensburg, 93040 Regensburg, Germany

ARTICLE INFO

Article history:

Received 17 January 2017

Received in revised form 26 April 2018

Accepted 1 May 2018

Available online 28 May 2018

Communicated by A. Blumberg

Keywords:

Beilinson regulator

K-theory

Absolute Hodge cohomology

Ring spectra

Motivic homotopy theory

ABSTRACT

We prove that the Beilinson regulator, which is a map from K -theory to absolute Hodge cohomology of a smooth variety, admits a refinement to a map of E_∞ -ring spectra in the sense of algebraic topology. To this end we exhibit absolute Hodge cohomology as the cohomology of a commutative differential graded algebra over \mathbb{R} . The associated spectrum to this CDGA is the target of the refinement of the regulator and the usual K -theory spectrum is the source. To prove this result we compute the space of maps from the motivic K -theory spectrum to the motivic spectrum that represents absolute Hodge cohomology using the motivic Snaith theorem. We identify those maps which admit an E_∞ -refinement and prove a uniqueness result for these refinements.

© 2018 Elsevier Inc. All rights reserved.

Contents

1. Introduction	42
2. Mixed Hodge complexes	44
3. The complex IDR and absolute Hodge cohomology	53

* Corresponding author.

E-mail address: georg.tamme@mathematik.uni-regensburg.de (G. Tamme).

¹ The authors are supported by the SFB/CRC 1085 *Higher Invariants* (Universität Regensburg) funded by the DFG.

4.	The category of motivic spectra	58
5.	The motivic K -theory spectrum	59
6.	The spectrum representing absolute Hodge cohomology	65
7.	The regulator	68
Appendix A.	Infinity categories and weak equivalences	74
Appendix B.	Representable functors and algebra structures	78
Appendix C.	Algebra localizations	83
References	85

1. Introduction

Let X be a smooth algebraic variety over \mathbb{C} . On the one hand, we can form its algebraic K -groups $K_*(X)$, which encode information about the symmetric monoidal category of vector bundles on X with respect to the direct sum. On the other hand, we have the Betti cohomology groups $H^*(X(\mathbb{C}), \mathbb{R})$, which carry a natural mixed \mathbb{R} -Hodge structure by [9]. In [1] Beilinson constructs a natural complex in the derived category of mixed \mathbb{R} -Hodge structures whose cohomology is the Betti cohomology with its mixed Hodge structure, and he defines absolute cohomology groups $H_{\text{abs. Hodge}}^*(X, \mathbb{R}(i))$ of X as Ext-groups of the Tate Hodge structure $\mathbb{R}(-i)$ and this complex. The absolute Hodge cohomology groups are the target of the Beilinson regulator, a natural homomorphism of graded groups

$$\text{reg} : K_*(X) \rightarrow \bigoplus_{i \in \mathbb{N}} H_{\text{abs. Hodge}}^{2i-*}(X, \mathbb{R}(i)) . \quad (1)$$

The tensor product of vector bundles induces a commutative ring structure on $K_*(X)$, and the \cup -product in the Betti cohomology of X provides a commutative ring structure on $\bigoplus_{i \in \mathbb{N}} H_{\text{abs. Hodge}}^*(X, \mathbb{R}(i))$. It is known that the regulator is a homomorphism of rings [15, 2.35].

Our main motivation for the present paper is the application of the regulator to a multiplicative version of differential algebraic K -theory as discussed in the series of papers [2, 5, 6]. For this, one needs a more refined version of the regulator map: The algebraic K -groups $K_*(X)$ are defined as homotopy groups of an algebraic K -theory spectrum $\mathcal{K}(X)$ and the multiplication on the K -groups is induced by an E_∞ -ring structure on this spectrum. Absolute Hodge cohomology on the other hand is defined as Ext-groups in the derived category of mixed Hodge complexes. We realize the absolute Hodge cohomology groups as the cohomology groups of a specific chain complex $\mathbf{IDR}(X)$ (Definition 3.3) consisting of differential forms. The usual wedge product of forms gives a multiplication on the chain level, i.e. it makes $\mathbf{IDR}(X)$ into a commutative differential graded algebra. Under the Eilenberg–MacLane equivalence H this commutative differential graded algebra induces an E_∞ -ring spectrum $H(\mathbf{IDR}(X))$ whose homotopy groups are the cohomology groups of $\mathbf{IDR}(X)$ and therefore the absolute Hodge cohomology groups of X . For the application we have in mind, it is an important question whether the regulator

Download English Version:

<https://daneshyari.com/en/article/8904697>

Download Persian Version:

<https://daneshyari.com/article/8904697>

[Daneshyari.com](https://daneshyari.com)