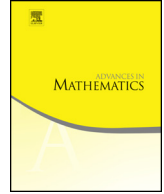




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Algebraic and o-minimal flows on complex and real tori [☆]



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ABSTRACT

We consider the covering map $\pi : \mathbb{C}^n \rightarrow \mathbb{T}$ of a compact complex torus. Given an algebraic variety $X \subseteq \mathbb{C}^n$ we describe the topological closure of $\pi(X)$ in \mathbb{T} . We obtain a similar description when \mathbb{T} is a real torus and $X \subseteq \mathbb{R}^n$ is a set definable in an o-minimal structure over the reals.

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1. Introduction

Let A be a complex abelian variety of dimension n , and let $\pi : \mathbb{C}^n \rightarrow A$ be its covering map. It follows from a theorem of Ax (see [1, Theorem 3]), that if $X \subseteq \mathbb{C}^n$ is an algebraic variety then the Zariski closure of $\pi(X)$ is a union of finitely many cosets of abelian subvarieties of A .

In [6,7], Ullmo and Yafaev attempt to characterize the *topological* closure of $\pi(X)$ in the above setting and also in the case that X is a set definable in an o-minimal expansion of the real field.

They prove a similar result to Ax’s for algebraic curves (see [6, Theorem 2.4]: if $X \subseteq \mathbb{C}^n$ is an irreducible algebraic curve then the topological closure of $\pi(X)$ in A is

$$\text{cl}(\pi(X)) = \pi(X) \cup \bigcup_{k=1}^m Z_k,$$

where each Z_k is a real weakly special subvariety of A , namely a coset of a real Lie subgroup of A . They conjecture that the same is true for algebraic subvarieties $X \subseteq \mathbb{C}^n$ of arbitrary dimension.

In this article we give a full description of $\text{cl}(\pi(X))$ when X is an algebraic subvariety of \mathbb{C}^n of arbitrary dimension and also when $X \subseteq \mathbb{R}^n$ is definable in an o-minimal structure over the reals and $\pi : \mathbb{R}^n \rightarrow \mathbb{T}$ is the covering map of a compact real torus.

As we show, the conjecture from [6] fails as stated (see Section 8) and we prove a modified version by showing that the frontier of $\pi(X)$ consists of finitely many families of real weakly special subvarieties. Our theorem holds for arbitrary compact complex tori and not only for abelian varieties.

Theorem 1.1. *Let $\pi : \mathbb{C}^n \rightarrow \mathbb{T}$ be the covering map of a compact complex torus and let X be an algebraic subvariety of \mathbb{C}^n . Then there are finitely many algebraic subvarieties*

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