Advances in Mathematics 333 (2018) 796–821

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Jacobi polynomials, Bernstein-type inequalities and dispersion estimates for the discrete Laguerre operator ☆

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A R T I C L E   I N F O

Article history:
Received 19 January 2018
Received in revised form 26 April 2018
Accepted 17 May 2018
Available online xxxx
Communicated by George Andrews

MSC:
primary 33C45, 47B36
secondary 81U30, 81Q05

Keywords:
Schrödinger equation
Dispersive estimates
Jacobi polynomials

A B S T R A C T

The present paper is about Bernstein-type estimates for Jacobi polynomials and their applications to various branches in mathematics. This is an old topic but we want to add a new wrinkle by establishing some intriguing connections with dispersive estimates for a certain class of Schrödinger equations whose Hamiltonian is given by the generalized Laguerre operator. More precisely, we show that dispersive estimates for the Schrödinger equation associated with the generalized Laguerre operator are connected with Bernstein-type inequalities for Jacobi polynomials. We use known uniform estimates for Jacobi polynomials to establish some

☆ Research supported by the Austrian Science Fund (FWF) under Grant No. P26060.

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https://doi.org/10.1016/j.aim.2018.05.038
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new dispersive estimates. In turn, the optimal dispersive decay estimates lead to new Bernstein-type inequalities.
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1. Introduction

To set the stage, for α, $\beta > -1$, let $w^{(\alpha,\beta)}(x) = (1-x)^\alpha(1+x)^\beta$ for $x \in (-1,1)$ be a Jacobi weight. The corresponding orthogonal polynomials $P_n^{(\alpha,\beta)}$, normalized by

$$P_n^{(\alpha,\beta)}(1) = \binom{n+\alpha}{n} = \frac{(\alpha+1)_n}{n!} \tag{1.1}$$

for all $n \in \mathbb{N}_0$ (see (1.21) for notation of Pochhammer symbols and binomial coefficients), are called the *Jacobi polynomials*. They are expressed as (terminating) Gauss hypergeometric series (1.22) by [42, (4.21.2)]

$$\frac{P_n^{(\alpha,\beta)}(x)}{P_n^{(\alpha,\beta)}(1)} = {}_2F_1\left(\begin{matrix}-n, n+\alpha+\beta+1\\ \alpha+1\end{matrix}; \frac{1-x}{2}\right). \tag{1.2}$$

They also satisfy Rodrigues' formula [42, (4.3.1), (4.3.2)]

$$\begin{aligned} P_n^{(\alpha,\beta)}(x) &= \sum_{k=0}^{n} \binom{n+\alpha}{n-k}\binom{n+\beta}{k}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k} && (1.3)\\ &= \frac{(-1)^n}{2^n n!}(1-x)^{-\alpha}(1+x)^{-\beta}\frac{d^n}{dx^n}\{(1-x)^{\alpha+n}(1+x)^{\beta+n}\}. && (1.4)\end{aligned}$$

Note that, by (1.3), $P_n^{(\alpha,\beta)}(x)$ is for given n a polynomial in x, α and β. Thus, if we don't need the orthogonality relations of the Jacobi polynomials, then we are not restricted by the bounds $\alpha, \beta > -1$.

The (squared normalized) L^2 norm of $P_n^{(\alpha,\beta)}$ is given by [42, (4.3.3)]

$$\begin{aligned}&\frac{\Gamma(\alpha+\beta+2)}{2^{\alpha+\beta+1}\Gamma(\alpha+1)\Gamma(\beta+1)}\int_{-1}^{1}|P_n^{(\alpha,\beta)}(x)|^2 w^{(\alpha,\beta)}(x)dx\\ &\qquad = \frac{n+\alpha+\beta+1}{2n+\alpha+\beta+1}\,\frac{(\alpha+1)_n(\beta+1)_n}{(\alpha+\beta+2)_n\, n!}.\end{aligned} \tag{1.5}$$

Jacobi polynomials include the ultraspherical (Gegenbauer) polynomials [42, (4.37.1)]

$$P_n^{(\lambda)}(x) := \frac{(2\lambda)_n}{(\lambda+1/2)_n}\, P_n^{(\lambda-\frac12,\lambda-\frac12)}(x), \tag{1.6}$$

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