Advances in Mathematics 333 (2018) 1293–1296

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Corrigendum

Corrigendum to “The six operations in equivariant motivic homotopy theory” [Adv. Math. 305 (2017) 197–279]

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ARTICLE INFO

Article history:
Available online 5 June 2018
Communicated by A. Blumberg

MSC:
14F42
55P91

Keywords:
Motivic homotopy theory
Equivariant homotopy theory
Algebraic stacks

The proof of the key Theorem 2.21 in [4] is incorrect. Towards the end of the proof it is claimed that $\mathcal{I}(v) \simeq \mathcal{J}(v)$, but in fact we only have an epimorphism $\mathcal{J}(v) \twoheadrightarrow \mathcal{I}(v)$, and it is trivial to obtain an example where the constructed scheme $\hat{V}$ fails to be flat at v over S; for instance, it can be arranged that $\hat{V} = V$.

The result is nevertheless true if we allow passing to an étale neighborhood of V in X, as we now explain. We fix a base scheme B and a flat finitely presented group scheme G

DOI of original article: https://doi.org/10.1016/j.aim.2016.09.031.
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https://doi.org/10.1016/j.aim.2018.05.011

over B. Given a G-scheme X and a G-invariant closed subscheme $Z \subset X$, we shall say that a G-morphism $f\colon X' \to X$ is a *G-equivariant étale neighborhood* of Z if it is locally finitely presented, if the induced map $Z \times_X X' \to Z$ is an isomorphism, and if f is étale at all points lying over Z.

Theorem A *(Correct version of Theorem 2.21). Suppose that B is affine and that G is linearly reductive. Let*

$$\begin{array}{ccc} X_Z & \overset{t}{\hookrightarrow} & X \\ \downarrow & & \downarrow{\scriptstyle p} \\ Z & \overset{s}{\hookrightarrow} & S \end{array}$$

be a cartesian square of G-schemes where X is affine and s is a closed immersion. Let $V \subset X_Z$ be a finitely presented G-invariant closed subscheme of X_Z that is smooth (resp. étale) over Z. Suppose that p is smooth at each point of V and that X has the G-resolution property. Then there exists an affine G-equivariant étale neighborhood $X' \to X$ of V and a finitely presented G-invariant closed subscheme $\hat{V} \subset X'$ lifting V such that $\hat{V} \to S$ is smooth (resp. étale) at each point of V.

Theorem 2.21 is cited five times in the rest of the paper, but Theorem A suffices in each case:

- In Remark 2.22: This remark should now be ignored.
- In Corollary 2.23: No changes are needed.
- In Corollary 2.24: No changes are needed.
- In Lemma 4.11: To use Theorem A, one has to replace $\hat{V}$ (hence $\tilde{V}$ and $\tilde{W}$) by an affine G-equivariant étale neighborhood of W; this does not affect the rest of the argument.
- In Theorem 4.18: When proving assertion $(*)$, after reducing to the case S small and affine, Lemma 4.16 allows us to replace X by any affine G-equivariant étale neighborhood of $t(Z)$. Given this, Theorem A applies in the same way.

To prove Theorem A, we need an equivariant version of Arabia's theorem on lifting locally free sheaves[1] along closed immersions [1, Théorème 1.2.3]:

Theorem B. *Suppose that B is affine and that G is linearly reductive. Let $s\colon Z \hookrightarrow X$ be a closed G-immersion between affine G-schemes and let $\mathcal{N}$ be a locally free G-module on Z. If X has the G-resolution property, there exists an affine G-equivariant étale neighborhood $X' \to X$ of Z and a locally free G-module $\mathcal{M}$ on X' lifting $\mathcal{N}$.*

1 Locally free sheaves are tacitly of finite rank in what follows.