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Commutators in groups of piecewise projective homeomorphisms ☆

José Burillo [a,*], Yash Lodha [b], Lawrence Reeves [c]

[a] *Departament de Matemàtiques, UPC, C/Jordi Girona 1-3, 08034 Barcelona, Spain*
[b] *Department of Mathematics, EPFL, Lausanne, Switzerland*
[c] *School of Mathematics and Statistics, University of Melbourne, Victoria, Australia*

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ABSTRACT

In [7] Monod introduced examples of groups of piecewise projective homeomorphisms which are not amenable and which do not contain free subgroups, and in [6] Lodha and Moore introduced examples of finitely presented groups with the same property. In this article we examine the normal subgroup structure of these groups. Two important cases of our results are the groups H and G_0. We show that the group H of piecewise projective homeomorphisms of $\mathbb{R}$ has the property that H'' is simple and that every proper quotient of H is metabelian. We establish simplicity of the commutator subgroup of the group G_0, which admits a presentation with 3 generators and 9 relations. Further, we show that every proper quotient of G_0 is abelian. It follows that the normal subgroups of these groups are in bijective correspondence with those of the abelian (or metabelian) quotient.

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* Corresponding author.
E-mail addresses: pep.burillo@upc.edu (J. Burillo), yash.lodha@epfl.ch (Y. Lodha), lreeves@unimelb.edu.au (L. Reeves).

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0. Introduction

In [7] Monod proved that the group H of piecewise projective homeomorphisms of the real line is non-amenable and does not contain non-abelian free subgroups. This provides a new counterexample to the so called von Neumann–Day problem [8,2]. In fact, Monod introduced a family of groups $H(A)$ for a subring A of $\mathbb{R}$. In the case where A is strictly larger than $\mathbb{Z}$, they were all demonstrated to be counterexamples. The group H is the case in which $A = \mathbb{R}$.

The subgroups G_0 and G of H were introduced by Lodha and Moore in [6] as finitely presented counterexamples. The groups G_0 and G share many features with Thompson's group F. They can be viewed as groups of homeomorphisms of the Cantor set of infinite binary sequences, and as groups of homeomorphisms of the real line. They admit small finite presentations, and symmetric infinite presentations with a natural normal form [6, 5]. Further, they are of type F_∞ [5]. Viewed as homeomorphisms of the Cantor set, the elements can be represented by tree diagrams.

Thompson's group F satisfies the property that F' is simple, and every proper quotient of F is abelian [1]. In this article we examine the normal subgroup structure, and in particular the commutator subgroup structure of G, G_0, and $H(A)$ for a subring A of $\mathbb{R}$, and obtain properties similar to F. We prove the following.

Theorem 1. *Let A be a subring of $\mathbb{R}$. If A has units other than ± 1, then:*

(1) $H(A)' \neq H(A)''$.
(2) *$H(A)''$ is simple.*
(3) *Every proper quotient of $H(A)$ is metabelian.*

If the only units in A are ± 1, then:

(1) *$H(A)'$ is simple.*
(2) *Every proper quotient of $H(A)$ is abelian.*
(3) *All finite index subgroups of $H(A)$ are normal in $H(A)$.*

We show the following for the finitely presented groups G_0 and G (defined in Section 1).

Theorem 2. *The group G_0 satisfies the following:*

(1) *G_0' is simple.*
(2) *Every proper quotient of G_0 is abelian.*
(3) *All finite index subgroups of G_0 are normal in G_0.*

The group G satisfies the following: