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Nonvanishing of central L-values of Maass forms

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ABSTRACT

With the method of moments and the mollification method, we study the central L-values of GL(2) Maass forms of weight 0 and level 1 and establish a positive-proportional nonvanishing result of such values in the aspect of large spectral parameter in short intervals, which is qualitatively optimal in view of Weyl's law. As an application of this result and a formula of Katok–Sarnak, we give a nonvanishing result on the first Fourier coefficients of Maass forms of weight $\frac{1}{2}$ and level 4 in the Kohnen plus space.

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1. Introduction

Nonvanishing of central L-values and their derivatives of automorphic forms is an important research topic, due to the connection between such values and various aspects of mathematics, such as arithmetic geometry, spectral deformation theory, and analytic number theory. The combination of the method of moments and the mollification method, popularized by Iwaniec–Sarnak [12], has been a very fruitful approach in yielding positive-proportional nonvanishing results on central L-values and their derivatives in a family of automorphic forms (see, e.g., [1], [11], [30], [15], [16], [17], [28], [18], [4], [14], [24], [22], and others). Along this direction we address the case of GL(2) Maass forms. Specifically, we study the (mollified) moments of the L-functions of the Hecke–Maass forms of weight 0 and level 1 at the central point of the critical strip, and establish a positive-proportional nonvanishing result of such values in short intervals of the spectral parameters (Theorem 1). As an application, this result and a formula of Katok–Sarnak (see (5)) imply a strong nonvanishing result (Theorem 2) of the first Fourier coefficient of Maass forms in the Kohnen plus space of weight $\frac{1}{2}$ and level 4.

Let $S_0(1)$ be the space of Maass cusp forms of weight 0 and level 1 and pick an orthonormal basis $\{u_j\}$ of Hecke–Maass forms of $S_0(1)$, where each u_j has Laplace eigenvalue $\frac{1}{4} + t_j^2$ ($t_j \geq 0$). (See § 2.1 for a brief review of Maass forms.) Our main result is the following

Theorem 1. *Fix $\eta \in (0,1)$ and let T and M be large parameters such that $T^\eta < M < T(\log T)^{-1}$. We have*

$$\#\left\{t_j \mid |t_j - T| \leq M, L(\tfrac{1}{2}, u_j) > 0\right\} \gg TM.$$

By Weyl's law (see [31] and [5])

$$N(T) := \#\{j \mid t_j \leq T\} = \frac{1}{12}T^2 - \frac{1}{2\pi}T\log T + c_0 T + O(T(\log T)^{-1}),$$