

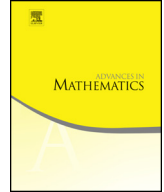


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## Gravity formality

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### ABSTRACT

We show that Willwacher’s cyclic formality theorem can be extended to preserve natural Gravity operations on cyclic multivector fields and cyclic multidifferential operators. We express this in terms of a homotopy Gravity quasi-isomorphism with explicit local formulas. For this, we develop operadic tools related to mixed complexes and cyclic homology and prove that the operad  $M_{\circlearrowleft}$  of natural operations on cyclic operators is formal and hence quasi-isomorphic to the Gravity operad.

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## 0. Introduction

The original deformation quantization problem aims to obtain a formal deformation of the associative product of functions of a Poisson manifold  $M$ , called a *star product*. The space governing such deformations is essentially the Lie algebra of multidifferential operators  $D_{\text{poly}}$ , the smooth version of the Hochschild complex of the algebra  $C^\infty(M)$ . In his celebrated paper [21], Kontsevich showed that the Lie algebra  $D_{\text{poly}}$  is formal i.e., it is quasi-isomorphic to its homology, the Lie algebra of multivector fields  $T_{\text{poly}}$ . His proof involves the construction of the “formality morphism”, a homotopy quasi-isomorphism of Lie algebras

$$U: T_{\text{poly}} \rightarrow D_{\text{poly}},$$

with explicit local formulas depending on integrals over configurations of points and expressed in terms of graphs. This result solves the deformation quantization problem by establishing a correspondence between formal Poisson structures and star products (bijective up to gauge equivalence).

Kontsevich’s result, however, ignores the richer structures existent on  $T_{\text{poly}}$  and  $D_{\text{poly}}$ . Let now  $M$  be an oriented  $D$ -dimensional manifold with a fixed volume form  $\omega$ . The pull back of the de Rham differential via contraction with  $\omega$  endows the space  $T_{\text{poly}}$  with the structure of a BV algebra. On the other hand, there is a natural action of the cyclic group of order  $n + 1$  on  $D_{\text{poly}}^n$  given by “integration by parts” which, after the cyclic Deligne’s conjecture (see [18]), induces a natural  $\text{BV}_\infty$  algebra structure on  $D_{\text{poly}}$ . The natural question to ask is whether Kontsevich’s formality morphism can be extended to a  $\text{BV}_\infty$  quasi-isomorphism. Tamarkin [26,16] constructed a non-explicit  $\text{Ger}_\infty$  (homotopy Gerstenhaber) quasi-isomorphism  $T_{\text{poly}} \rightarrow D_{\text{poly}}$  depending on a solution of Deligne’s conjecture whose underlying  $\text{Lie}_\infty$  morphism was later shown by Willwacher [31] to be homotopy equivalent to Kontsevich’s map if one uses the Alekseev–Torossian associator to construct a solution to Deligne’s conjecture. Furthermore, Willwacher shows that the original formality morphism can be strictly extended to a  $\text{Ger}_\infty$  morphism. The full extension to the BV setting was given by the first author [2] who constructed a  $\text{BV}_\infty$  quasi-isomorphism  $T_{\text{poly}} \rightarrow D_{\text{poly}}$  with explicit local formulas depending on integrals over configurations of framed points. One advantage of incorporating these richer structures into the discussion is that we may now view the algebraic operations as being parametrized by geometric objects, namely by the moduli spaces of genus zero surfaces with parametrized boundary components.

The subspace of cyclic invariants of  $D_{\text{poly}}$ , denoted by  $D_{\text{poly}}^\sigma := \bigoplus_{n \geq 0} (D_{\text{poly}}^n)^{\mathbb{Z}_{n+1}}$ , is preserved by the Lie bracket and the Hochschild differential. The differential graded Lie algebra  $D_{\text{poly}}^\sigma$  is associated to a different deformation problem, namely the construction of *closed star products*. This led to the conjecture of an analogous formality statement, the “cyclic formality conjecture” [25]. Let  $\text{div}_\omega: T_{\text{poly}}^\bullet \rightarrow T_{\text{poly}}^{\bullet-1}$  be the divergence operator

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