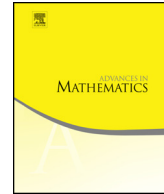




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Strict n -categories and augmented directed complexes model homotopy types



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ABSTRACT

In this paper we show that both the homotopy category of strict n -categories, $1 \leq n \leq \infty$, and the homotopy category of Steiner's augmented directed complexes are equivalent to the category of homotopy types. In order to do so, we first prove a general result, based on a strategy of Fritsch and Latch, giving sufficient conditions for a nerve functor with values in simplicial sets to induce an equivalence at the level of homotopy categories. We then apply this result to strict n -categories and augmented directed complexes, where the assumption of our theorem was first established by Ara and Maltsiniotis and of which we give an independent proof.

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Introduction

The homotopy theory of small categories was born with the introduction by Grothendieck of the nerve functor in [6], allowing the definition of weak equivalences in Cat , the category of small categories: a functor is a weak equivalence precisely when its nerve is a simplicial weak equivalence.

The first striking result of this theory appears in Illusie's thesis [8] (who credits it to Quillen) and states that the induced nerve functor induces an equivalence at the level of the homotopy categories. A careful study of the subtle homotopy theory of small categories by Thomason [12] led him to show another important result: the existence of a model structure on Cat which is Quillen equivalent to the Kan–Quillen model structure on simplicial sets. This implies that small categories and simplicial sets are not only equivalent as homotopy categories, but actually as $(\infty, 1)$ -categories.

Similarly Chiche [4], using ideas of del Hoyo [7], has shown that strict 2-categories model homotopy types. Moreover, Ara and Maltsiniotis [1] provided an abstract framework in which a Quillen equivalence between strict n -categories and simplicial sets can be established, using the Street nerve, as long as some conditions are satisfied; they also showed that these conditions hold for small 2-categories. One of these conditions, which they called condition (e), states that the unit morphism of the adjunction given by the Street nerve is a weak equivalence when evaluated at the nerve of a poset. In a subsequent paper [2] they proved in full generality this condition (e), making an important use of the augmented directed complexes introduced by Steiner [10], which act as a linearisation of ∞ -categories.

In the same article, they also conjectured that for $1 \leq n \leq \infty$ the Street nerve induces an equivalence of homotopy categories between strict n -categories and simplicial sets, and that the same holds for augmented directed complexes with their own nerve functor. The aim of this paper is to prove these conjectures.

We consider a general framework with a nerve $U: \mathcal{C} \rightarrow \hat{\Delta}$ induced as a right adjoint of a left Kan extension F of a cosimplicial object of $\mathcal{C}$ along the Yoneda embedding of Δ , where $\mathcal{C}$ is a cocomplete category, Δ is the category of simplices and $\hat{\Delta}$ is the category of simplicial sets. Using the properties of simplicial subdivision and adapting a strategy developed in an article by Fritsch and Latch [5], a short proof that U induces an equivalence at the level of homotopy categories can be given, as long as it satisfies an appropriate version of condition (e). This condition says that the unit $\eta: 1_{\hat{\Delta}} \rightarrow UF$ is a weak equivalence on the nerves of posets, that is for every poset E , the morphism of simplicial sets $N_1(E) \rightarrow UF(N_1(E))$ is a weak equivalence, where $N_1: Cat \rightarrow \hat{\Delta}$ denotes the usual nerve. As a first example, we give a short proof of the well-known result stating that (ordered) simplicial complexes model homotopy types. More interestingly, Ara and Maltsiniotis [2] have established condition (e) for both the nerve $N_n: n\text{-}Cat \rightarrow \hat{\Delta}$ of strict n -categories, $1 \leq n \leq \infty$, and the nerve $N: \mathcal{C}_{da} \rightarrow \hat{\Delta}$ of augmented directed complexes. We can thus apply our theorem to these cases and get a proof of the above conjectures. Nevertheless, we give an independent proof of the condi-

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