Advances in Mathematics 331 (2018) 874–907

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Super-potentials, densities of currents and number of periodic points for holomorphic maps

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A R T I C L E I N F O

Article history:
Received 15 October 2017
Accepted 29 March 2018
Communicated by the Managing Editors

Dedicated to Professor Lê Tuân Hoa on the occasion of his sixtieth birthday

MSC:
32U40
32H50
37F05

Keywords:
Super-potential
Tangent current
Periodic point
Green currents

A B S T R A C T

We prove that if a positive closed current is bounded by another one with bounded, continuous or Hölder continuous super-potentials, then it inherits the same property. There are two different methods to define wedge-products of positive closed currents of arbitrary bi-degree on compact Kähler manifolds using super-potentials and densities. When the first method applies, we show that the second method also applies and gives the same result. As an application, we obtain a sharp upper bound for the number of isolated periodic points of holomorphic maps on compact Kähler manifolds whose actions on cohomology are simple. A similar result still holds for a large class of holomorphic correspondences.

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https://doi.org/10.1016/j.aim.2018.04.005
0001-8708/

Equilibrium measure
Correspondence

1. Introduction

Positive closed currents are a fundamental tool in complex analysis, algebraic geometry, differential geometry and complex dynamics. Bi-degree $(1,1)$-currents and their intersections were intensively studied and have had many applications while arbitrary bi-degree currents are much less understood, see Bedford–Taylor [1], Demailly [5,6] and Fornaess–Sibony [20]. In [4, p. 16], Demailly posed the problem to develop a theory of intersection for positive closed currents of higher bi-degree. Sibony and the first author have recently introduced, in a series of articles [14,15,17], two different approaches to this problem in the context of compact Kähler manifolds. Several applications in complex dynamics and foliation theory have been obtained as well.

Let X be a compact Kähler manifold of dimension k. Let T and S be positive closed currents of bi-degree respective (p,p) and (q,q) on X. We will briefly recall the notions of super-potentials and densities of positive closed currents. We then describe the two different approaches to define the wedge-product (intersection) of T and S. Our first aim is to prove that when both methods apply we obtain the same current. This will allow us to unify the advantages of both methods. We also consider a domination principle for super-potentials and apply our study to bound the number of isolated periodic points of holomorphic maps or correspondences.

The notion of super-potentials of positive closed currents were introduced in [14,15]. We refer to these references for details. The starting point is that the pluripotential theory is well developed for positive closed currents of bi-degree $(1,1)$ thanks to the notion of plurisubharmonic (p.s.h. for short) functions. More precisely, if T is a positive closed $(1,1)$-current, then we can write locally $T = dd^c u$ in the sense of currents, where u is a p.s.h. function, $d^c := \frac{i}{2\pi}(\partial - \overline{\partial})$ and $dd^c = \frac{i}{\pi}\partial\overline{\partial}$. Since the function u is everywhere defined, if u is integrable with respect to the trace measure of S (we will say "with respect to S" for short), then one can define uS in the sense of currents and define $T \wedge S := dd^c(uS)$. It is not difficult to check that the definition is independent of the choice of u because u is unique up to a pluriharmonic function. When T is of higher bi-degree, we still can write locally $T = dd^c U$ but the potential U does not satisfy the above properties of p.s.h. functions and one cannot consider their wedge-product in the same way.

Super-potentials are functions which play the role of quasi-potentials for positive closed currents of arbitrary bi-degree. For simplicity, we will not introduce this notion in full generality but limit ourselves in the necessary setting. Let $\mathcal{D}_q(X)$ (or $\mathcal{D}_q$ for short) denote the real vector space spanned by positive closed (q,q)-currents on X. Define the $*$-*norm* on this space by $\|R\|_* := \min(\|R^+\| + \|R^-\|)$, where $R^\pm$ are positive closed (q,q)-currents satisfying $R = R^+ - R^-$ and $\|\ \|$ denotes the mass of a current. We consider this space of currents with the following *topology*: a sequence $(R_n)_{n\geq 0}$ in $\mathcal{D}_q(X)$